Slide Rule Accuracy vs. Precision

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Version, 8/10/99

Introduction

Patently, the most widely published slide rule instruction manual was the book by Kells, Kern and Bland which accompanied every K+E Log Log Duplex slide rule ever manufactured. This book, in its beginning pages, usually "Section 3," has a paragraph entitled "Accuracy of the Slide Rule." That this very paragraph discusses slide rule precision, not slide rule accuracy, shows that a gross error was propagated for decades by the authors engaged by K+E to write their manuals and, further, that no editor or stylist ever corrected the paragraph. Because there is a world of difference between the meanings of the two words "accuracy" and "precision" this present paper is written to clarify the meanings of the contructs of accuracy and precision, and will do so by means of examples which are related to scales usage.

This paper will also present a discussion of both initial manufacture and longevity effects on slide rule accuracy; a discussion of accuracy and precision characteristics of slide rules shorter and longer than ten inches as compared with the accuracy and precision of ten inch rules; and, in an Appendix, the paper will present a twelve-level slide rule accuracy evaluation sequence for 10" LogLog Duplex slide rules, a sequence developed through an extensive program of slide rule evaluations.

The Laboratory Scale Experiment

In an Elementary Physics course laboratory, a class of 30 students, arranged in teams of two, are given the following materials and assignment:

Each team is handed a 12" steel scale, a piece of white bond paper, a sharpened pencil, and a 10x magnifying glass; the steel scales are engine divided in 1/100 inch intervals.

The student teams are instructed to make two marks an arbitrary distance apart of their sheets of paper.

The student teams are then instructed to use their steel scales to measure the distance between their two marks 100 times, using the magnifying glass, recording each measurement made in tabular form on their data sheet, and to alternate measuring between the two team members. The distance measurements are to be made to the nearest 1/100 of an inch, with interpolations to be made should the pencil mark lie between two adjacent l/100 inch graduations of the steel measuring scale.

When the series of 100 measurements is complete, the student teams are to compute the average and the standard deviation of their individual sets of data, and then hand in their results.

When, at the following session of the Elementary Physics laboratory, the student teams are handed back their measurement results papers from the previous laboratory session, the students find that each team's results, i.e., the average and the standard deviation computed, are graded with a big red "F" for Fail. Naturally, the students want to know why their very careful work had universally been graded Fail.

To answer the students' questions, the teacher handed back to each student team one of the steel scales that they had used at the previous laboratory session. The students were directed to study the fine writing at the left end of the rules, which writing stated:

"Linear shrink: steel, puddled, l:50"

The teacher explained that, although the scales looked like fine, accurate, engraved steel rules, in fact these steel rules, marked "12", were actually 12.24" long! In other words, a l2" measurement made with the rule would be about a l/4" longer than 12", that a 6" measurement made with the rule would be about an l/8" longer than 6", and so forth. The teacher further explained that these rules were used to size patterns for sand molding of puddled steel alloy, and that castings made with that alloy shrink one part in 50 in every direction upon cooling. Thus the pattern created for the casting using this alloy would correspondingly have to be made one part in 50 larger in every direction in order to assure that the cooled casting would be of the desired dimensions.

Then to drive home hard the pivotal point of the entire exercise, the teacher lectured the students thus:

Use of the shrink rules to measure distances in actual inches and fractions of inches, down to l/l00" and, further, down to an estimated l/l000 of an inch by interpolation, was totally erroneous, since the shrink rules could be counted on to make measurements, say, of a 12" distance to only ¬" scale intervals, not l/100" or, more ridiculously, to l/l000" by interpolative estimates.

Thus the shrink rules were precise, because measurements made with the rules could be determined to within 1/l00 inch, and interpolations could be made to an estimated l/l000 inch. But for making true measurements, e.g., the measured distance between the marks made with sharpened pencil during the experiment, the rules were not accurate. In conclusion, the teacher stated, the rules appeared to be accurately made, but the rules were not accurate for measuring actual distances, the rules were only precise.

Other Accuracy vs. Precision Examples

Having presented the Laboratory Scale Experiment findings, the following examples will serve to further demonstrate the total, and absolute, difference in the meanings of the two distinct constructs: accuracy, and precision:

A gas tank gage in an automobile has a finely divided scale which can be used to read to the nearest 1/10 gallon. However, unbeknownst to the operator of the vehicle, a miscreant has secretly bent the needle of the gage at a point near the needle's pivot, a point that is hidden by the fascia of the instrument panel. The miscreant who bent the gage needle arranged the bend so that when the needle showed the gas tank as being "Full," the tank would actually be halffull. The gage then becomes an instrument that is precise, but that is woefully inaccurate.

A watch dial is graduated in l/5th of a second intervals between each minute mark. Thus the watch is precise. But unbeknownst to the person using the watch to observe the time, the watch is five minutes slow; reading a time to the nearest 1/5th second with this watch, while being precise, is ridiculous, because the time reading is five whole minutes away from the true time - the watch is inaccurate.

The Constructs of Accuracy and Precision as Applied to Log Log Duplex Rules Contrary, then, to what Kells, Kern and Bland stated in every edition of the K+E instruction book, the readings the authors describe relate only to precision, i.e., the scale intervals that permit a user to read or set the rule to three or more places. Having the scale properties of precision states nothing about, and has no relationship whatever to, the properties of accuracy of the rule.

The accuracy of a slide rule has, at the time of manufacture, everything to do with how the engraving or printing of all of the scale graduations corresponds with the true mathematically computed positions of every graduation on the rule. Assuming then for the moment that a particular slide rule was accurately laid down at the time of manufacture, nothing specific can be said about the effects on that rule's accuracy down through time; those effects can include not only damage and abuse, but also in the case of a wood or paper rule, shrinkage or expansion, non-uniformly in a single direction or differentially in numerous directions throughout the entire volume of the rule.

In the case of the Log Log Duplex rule there is of course the all-important consideration of transfer of calculations from front to rear and from rear to front sides of the rule. Thus, in a Duplex rule the accuracy-damaging effects of time are potentially greatly enhanced because of the two-sided referencing that must be done with that style of rule, even if it is assumed that the Duplex rule was laid down accurately, both sides, and both sides in registry, at the time of manufacture.

Additional Accuracy-Limiting Factors in LogLog Duplex Rules When a rule is manufactured, the wood may not have been properly aged, and so the body or slide or both may warp, either in a single curve, or in a wavy curve, or the slide portion may warp differently than the body portions. In the latter two cases, the slide at various points along the mating edges, will lie either above or below the adjacent surface of the body, leading to parallax errors on reading and on setting, even if all of the rule's graduations were accurately laid down during manufacture. A rule can also become curved, one wave, multiple waves, differential waves, through bad storage or careless handling, or from warping that occurs over time; the limiting parallax effects abovedescribed also apply under these circumstances.

Either at the time of manufacture, or through aging, some of the body or slide edges may lose planar flatness, and flare out at some or all points along the body mating edges, or at the slide edges, or at all four mating edges, body and slide. This flaring-of-edges effect introduces parallax errors on reading and on setting.

It may prove impossible to bring front and rear cursor hairlines into perfect coincidence while at the same time bringing the pair of hairlines into perfect registration with the front and rear sides of the rule. The usual import of this impossibility, should it arise, is that the front and rear sides were either not in registration at the time of manufacture or the front and rear sides through time have proceeded out of overall registration.

Another accuracy-limiting cursor effect is related to the fit of the cursor to the slide rule body. Even if the cursor hairlines are in perfect coincidence front-to-rear, if the cursor is slightly loose on the body in the transverse direction, having some slack in that direction, then it is possible that:

a. The cursor can become angled with respect to the surface of the body, causing inaccurate readings from frontto-rear because the front-to-rear axis of the hairlines is not perpendicular to the body of the rule

b. The cursor may shift position when the rule is flipped over to utilize the other side of the rule body.

c. The cursor may lie fully flat on one side of the rule, causing the hairline of the cursor on the other side of the rule to be too high above the surface of the other side of the rule, leading to parallax errors in accuracy of reading and setting.

The cursor cannot be so tightly fitted on the body of the rule so as to be capable of being moved only with difficulty, yet the optimum free play of the cursor in the transverse position can be afforded with only a few thousandth's of a inch of transverse movement. However, having this near-perfect fit of the cursor means that any dirt that gets under the cursor windows must be removed; this removal of dirt can be accomplished easily by use of triangularly shaped, slightly moistened, slips of 20 lb. white paper, where the tip of the paper triangle is introduced under the cursor window, and then the cursor is slid back and forth atop the wider portions of the paper triangle.

There must be minimum gap widths between the mating scale edges, for if these gaps are too wide, there will be accuracy errors on reading and setting the rule. Gap width can be a function of maladjustment of the adjustable stator, but due to possible differential shrinkage and expansion of a rule through time, it may be impossible to reduce the gap by binding down with the adjustable slider without locking the slider in place.

Some have suggested to this writer that a slide rule might expand or contract along its length direction in such a way that it, if originally accurately laid down, will remain accurate. Leaving out Pickett metal rules, K+E, Dietzgen and Hemmi rules are all made of wood. Hemmi rules are made of a superior and more stable wood, bamboo, than the wood, mahogany, of which K+E and Dietzgen rules are made. Wood is an non-homogenous material and there is no reason why wood, on expansion or on contraction, would do so in an absolutely linear and uniform manner. If such were indeed possible, the rule at every point along its length, body and slide, both sides, would have to expand or contract with a uniformity of l/l000 of an inch, a certain impossibility.

To make realistic, but at the same time totally impracticable, the linear expansion and/or contraction suggestion, the rule would have to be constructed from heavy bars of platinumiridium alloy, an alloy having an exceedingly low coefficient of linear expansion or contraction. Until October 1960, the international meter was defined as the distance between two marks on a platinum-iridium bar housed in Paris. To present an idea of the level of accuracy involved with the standard of length, in October 1960, by international agreement, the meter was redefined to be l,650,763.73 wavelengths in vacuo of the orange-red spectral line of krypton 86.

Another effect that contributes to the potential masking of rule inaccuracies lies in the thickness of the graduations. Pickett rules in general have thicker graduations than Hemmi, K+E and Dietzgen rules. When the writer has conducted evaluations of Pickett rules, many have been noted to be inaccurately printed, but some Pickett rules that have been found to be accurate do meet the higher Levels (see Appendix) of accuracy through the agency of somewhat too-thick graduation lines. The writer has observed that the graduations of 1945 Hemmi

rules are thinner than the graduations found on the last-produced 1975 Hemmi rules.

Visual Acuity and the Slide Rule

The resolving power of the human eye is related to the visual angle subtended by the finest detail that the eye can distinguish. However, it is a property of the human eye, a property long made use of in optical devices such as split-image rangefinders, that the eye can distinguish line objects and the coincidence of or the lack of coincidence of line objects, at visual angles far less than those at the limit of the resolving power of the eye.

For example, one can easily see a distant telephone or power line although the visual angle subtended by the distant line object is much smaller than the visual angle subtended at the resolving power limit of the eye. One can distinguish at a distance if one line is close to, but not touching, another line.

The line-distinguishing property of the human eye makes facile the reading and setting of a slide rule, since the eye can work well with critical line alignment or line nonalignment; these being the visual tasks involved with slide rule calculations.

Slide Rules Shorter Than 10"

"Pocket" slide rules of the LogLog Duplex design have 5" scale lengths. Even if such a rule is accurately laid down, there are two effects which serve to limit the accuracy of a 5" rule as compared with a l0" rule:

a. The thickness of the graduation lines on the 5" rule cannot be less than the thickness of the graduation lines found on the 10" rule, while, logically, the graduations on a 5" rule should be 1/2 the thickness of the graduations on a l0" rule. As discussed above in this paper, too-thick graduation lines serve to mask inaccuracies, and thus lead to errors on setting and on reading.

b. The 5" rule is less precise than a l0" rule, since the 5" rule is not as finely divided as a 10" rule, necessarily so, as otherwise, the 5" rule's scales would be rendered useless through overcrowding.

Slide Rules Longer Than 10"

Examples of rules longer than 10" include the 20" Log Log Duplex rule, certain cylindrical rules, large circular rules, classroom wall demonstration rules, and the multiply-staved Thacher rule. It should be clear from this paper that rules longer than 10" can certainly provide more precision of setting and of reading, but again it will here be reemphasized that the scale properties of precision has no relationship whatever to the properties of accuracy of the rule.

If, for example, on a 10" rule, 1.01 and 9.95 can each be set on a graduation line, and if, for example, on 20" rule, 1.005 and 9.975 can each be set on a graduation line, there is absolutely no warranty that the increase of precision afforded by the 20" rule due to the increased fineness of the graduations on the 20" rule will result in more accurate calculations with the 20" rule than can be made with the 10" rule. This is because longer rules made of wood or paper could not be manufactured with greater accuracy than a 10" rule, and rules longer than 10" cannot withstand the effects of longevity, namely, differential expansion and contraction, warping, edgeflaring, as well as can a 10" rule. The costs of

manufacturing an accurate rule longer than 10" would far exceed the costs of manufacturing an accurate 10" rule.
It is well known in one of the scale procedures of highest accuracy, the field of ruling the lines of diffraction gratings, that the lengthy engraving machine lead screw is the single most extraordinarily costly element of the entire machine. Correspondingly, if a manufacturer set out to make accurate slide rules longer than 10", he doubtless would not utilize a wooden base for the slide rule body; also, his engraving machines would have to be crafted to be accurate over a distance of a least twice the length of a l0" rule, radically increasing the costs of the machines and correspondingly the sales prices of long accurate rules.

All this is not to say that all 10" rules were all accurately made, in fact, there is no proof that very many 10" slide rules at all were accurately made. What we do have in the way of proof, however, is the converse proof, made inferentially, through this writer's exhaustive and continuing program of slide rule evaluations, being as follows:

The fact that the writer has identified several near perfect and several fully perfectly accurate 10" Log Log Duplex slide rules, albeit many, many years having passed since these identified slide rules were manufactured, means that at one time or another the machinery for producing the 10" scale length duplex slide rule was capable of producing a totally accurate rule.

Obviously, it is only through unknown circumstances that a slide rule that originally was accurately manufactured would present itself today as still being an accurate slide rule.

Definitions

For scale readings and/or settings, as obtain in using slide rules, then:

a. To how many places can the scale be read or set is the measure of precision of the scale.

b. How close a reading or setting is to the true value of the number is the measure of the accuracy of the scale.

In Conclusion

The 10" Log Log Duplex slide rule is the optimum design for a slide rule, for that design is the single slide rule design that at the same time provides:

a. Potential for overall accuracy.

b. Two-sided design that allows for openness of scales layout, front and rear, and that makes complex calculations involving trigonometric, log, ln, reciprocal, square root, cube root, and exponential functions easy to accomplish, all in a compact package that can be easily grasped and that balances well in the hand.

As the single function of a slide rule is to enable the user to make accurate calculations, and as accurate calculations can only be made using a slide rule that is accurate, then it is this writer's opinion that an accurate 10" Log Log Duplex slide rule is the premier slide rule and that any slide rule that is less than accurate, or is of any design or configuration other than 10" Log Duplex, is merely a curiosity.

Appendix: Accuracy Evaluation Sequence For 10" Log Log Duplex Slide Rules

This accuracy evaluation presented below increases greatly in difficulty as a candidate rule is sequenced through each succeeding Level.

Tools Required

8X optical loupe

Set of screwdrivers of proper tip width and tip sharpness Wooden or plastic small mallet for tapping slider and adjustable stator (can be smooth wooden handle of old-style small screwdriver)

Preliminary Evaluation

Warping - examine rule, end-on, for warping; if excessive (see present paper), reject the rule.

Flaring - examine mating edges of scales to note the extent of any flaring; if excessive (see present paper), reject the rule.

Cursor Fit - examine fit of cursor; if loose transversely (see present paper) reject the rule.

Slider/Stator Gaps - check the gap widths between mating edges (see present paper); if gaps cannot be uniformly minimized, reject the rule.

Level Rating Evaluation

Level l: Check C vs. D on front side.

Level 2: Check A vs. B on rear side.

Level 3: Check C and D vs. A and B.

Level 4: Check CF vs. DF.

Level 5: Check D vs. Sin.

Level 6: Check CF and DF vs. D and Sin.

Setting A: Align stators to slider.

Level 7: Check C vs. D vs. CF vs. DF.

Level 8: Check A vs. B vs. Sin vs. D.

Level 9: Check C vs. D vs. CF vs. DF vs. A vs. B vs. Sin vs. D.

Cleaning: Clean the slide rule following the methods of Bruce Babcock (Oughtred Journal, Vol. 2, No. 2, October 1993, p. 18); clean the underside of the cursor windows (see present paper).

Setting B: Align cursor hairline to body on front side.

Level 10: Check all front side scales, bottom to top of rule.

Setting C: Align cursor hairline to body on rear side.

Level 11: Check all rear side scales, bottom to top of rule.

Setting D: Align cursor to bring front side hairline into coincidence with rear side hairline, while at the same time taking care to maintain the front side alignment of Setting B as well as the rear side alignment of Setting C.

Level 12: Check all scales, front side to rear side, bottom to top of rule.