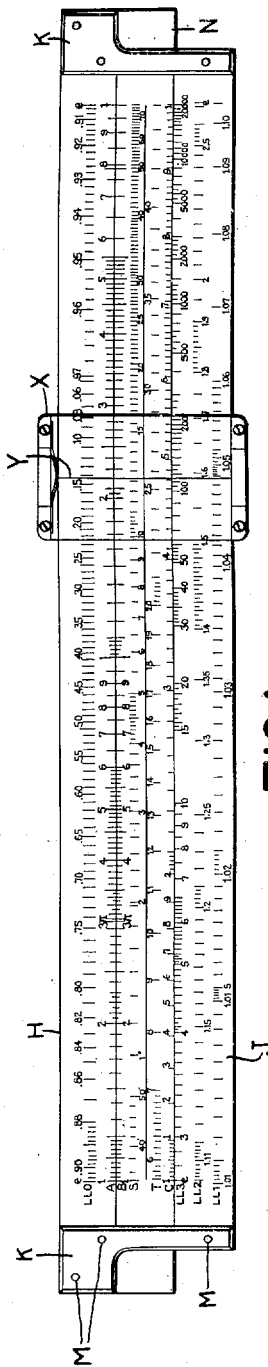FIG.1.
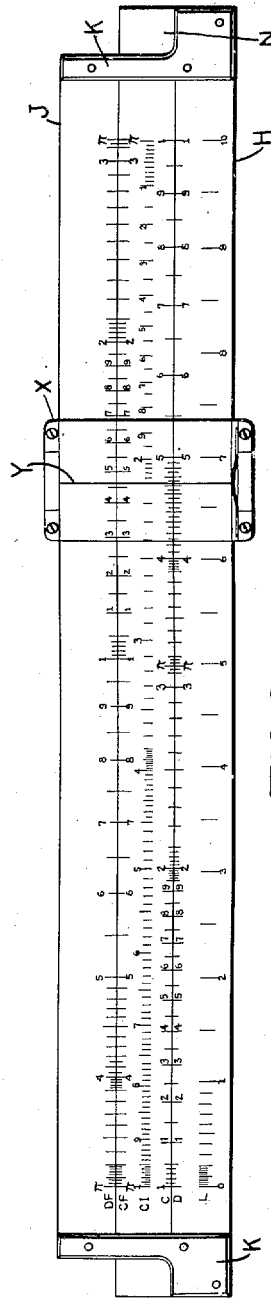
FIG.2.

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UNITED STATES PATENT OFFICE

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SLIDE RULE

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Keuffel & Esser Company, Hoboken, N. J., a
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Original No. 1,488,686, dated April 1, 1924, Serial
No. 566,340, June 6, 1922. Application for re-
issue April 7, 1938, Serial No. 200,711

2 Claims. (Cl. 235—70)

My invention relates to slide rules and more
particularly to slide rules adapted for the solution
of problems involving fractional powers and roots
as well as natural or hyperbolic logarithms, and
the novelty consists of the construction, adapta-
tion and arrangement of the parts as will be
more fully hereinafter pointed out.

The use of slide rules in many lines of business
as well as professions has increased rapidly with
the demand for quick and accurate solution of
mathematical problems because the processes of
business have changed, whereby instead of leav-
ing the solution of such problems to the conveni-
ence of a corps of poorly paid clerks such prob-
lems are now solved immediately by the use of
suitable slide rules adapted for the particular
purpose required. This has resulted in the de-
velopment and marketing of a large number of
different types of slide rules adapted for many
different purposes, but never has there been a
slide rule especially adapted for figuring powers
and roots of numbers less than unity whereby such
problems may be solved directly on the rule.

My slide rule solves this problem and furnishes
a means of solving problems involving numbers
less than unity, directly, without the use of re-
ciprocals.

Referring to the drawing: Figure 1 is a top
plan view of my slide rule; and Figure 2 is a top
plan view of the reverse side of my slide rule.

In the drawing bars H and J are rigidly secured
together by means of plates K which are riveted
thereto at M, so that a sliding bar N may be
mounted between said bars H and J so as to be
readily slidable longitudinally thereof. The slid-
able bar N has the usual tongue members on each
edge adapted to slide in the usual groove members
on the edges of the bars H and J contiguous to the
sliding bar B so that the sliding bar N is always
held in engagement between the bars H and J in
whatever position it occupies longitudinally
thereof. A runner or indicator X, transparent on
both faces and of usual construction, is mounted
over said bars H, J and N so as to be readily
moved into any position desired between the
plates K, and the runner X has a hair-line Y on
each side thereof.

On the front of my slide rule as shown in
Figure 1, the upper scale on the bar H is desig-
nated as LL0 and has graduations representing
the logarithms of co-logarithms of numbers less
than unity, in the illustrated embodiment, from
.05 to .97. The next scale on the bar H is
designated as A and contains a double standard
graduated logarithmic scale. The upper scale

on the sliding bar N is designated as B, and has
the same graduation as the scale A on the bar H.
A second scale on the sliding bar N is designated
as S, and has a graduated sine scale of degrees
and minutes from 34' to 90° and is used with
reference to scales A and B. The third scale on
the sliding bar N is designated as T and has a
graduated standard tangent scale with divisions
from 5° 43' to 45°. The fourth scale on the slid-
ing bar N is designated as C and has standard
graduated logarithmic divisions of full unit length
from 1 to 10. The upper scale on the bar J is
designated as LL3, and has graduations repre-
senting the logarithms of logarithms of the num-
bers from 2.7 to 22,000. The second scale on the
bar J is designated as LL2 and has graduations
representing the logarithms of logarithms of the
numbers from 1.105 to 2.7. The third scale on
the bar J is designated as LL1, and has gradua-
tions representing the logarithms of logarithms
of the numbers from 1.01 to 1.105.

On the back of my slide rule, as shown in
Figure 2, the scale shown on bar J is designated
as DF, and is a standard logarithmic scale of full
unit length the same as the C scale described
above, except that it is folded and has its index
at the centre. The upper scale on the sliding bar
N is designated as CF, and is identical with the
scale DF on bar J. The second scale on the
sliding bar N is designated as CI and is a standard
reciprocal logarithmic scale of full unit length
graduated from 10 to 1. The third scale on the
sliding bar N is designated as C and is a standard
logarithmic scale of full unit length identical with
C scale above described. The first scale on the
bar H is designated as D, and is the same as the
scale C on the sliding bar N. The second scale on
the bar H is designated as L and is a scale of equal
parts from 0 to 1 and is used to obtain common
logarithms when referred to scale D.

As has been above stated there have been slide
rules on the market for some time, with which
it is possible to solve problems involving frac-
tional powers and roots as well as natural or
hyperbolic logarithms. These slide rules of the
prior art had graduated logarithmic scales de-
signed as LL1, LL2 and LL3, the same as illus-
trated herein.

On these slides rules embodying the logarithmic
scales mentioned it was not possible to handle,
directly, roots and powers of numbers less than
unity, as the reciprocal of the quantity must be
first taken thereon, and this reciprocal then evalu-
ated in the integral scales and then the reciprocal
of the result taken in order to get the required

answer. This operation involved a number of additional operations, both mentally and mechanically, and increased the liability of error, as the correctness of the result depended not on the mechanical operation of the slide rule alone, but also upon the mental operation of the operator combined with the mechanical operation of the slide rule.

My slide rule has overcome this difficulty and made it possible to handle roots and powers of numbers less than unity so as to obtain the solution of examples in formulae involving powers and roots of numbers less than unity directly on my slide rule by mere mechanical operation of the same and without the use of reciprocals of the quantities handled, thereby eliminating any separate mental operation and the possibility of errors involved therein.

In the equation of the catenary curve and in hyperbolic functions, the expression e^{-x} occurs in which $e=2.7182+$ is the base of natural or Napierian logarithms, also in many formulae the logarithms to the base e are required or is to be taken as a factor.

By placing the reciprocal of e on scale LL0 in alignment with an index "1" of the regular logarithmic scale A, the values of e^{-x} can be at once read for all values of x within the range of the scale, without the setting of the slide N and also the co-logarithms to base e can be directly read on A.

In order to make this clearer I will give a few examples, showing the old method as heretofore used and showing how to solve the problems by the new method.

Example 1

Required $x=0.8^5$.

By the old method it is necessary to set the runner or indicator to .8 on scale C; read the reciprocal which is 1.25 on scale CI on the reverse side of the rule, keep this number 1.25 in mind and then set the right index of scale C to this number 1.25 on scale LL2 and opposite 5 on scale C read 3.05 on scale LL3, now keep 3.05 in mind and then set runner to this number 3.05 on C; and read the reciprocal .328 on CI which is the answer sought.

It will be noted that in this problem besides various operations of the slide and runner, it was necessary to take two readings mentally and reset the same on the slide rule, which may readily cause an error.

By my new method all that is necessary to solve the above example is to set the left index of scale B to 0.8 on scale LL0 and opposite 5 on scale B read answer .328 on scale LL0.

Example 2

Required $\log_e .635$.

Old method—set runner to .635 on C; read reciprocal 1.575 on CI, keep this number in mind and set runner to 1.575 on LL2; read .454 on D=$\log_e$ 1.575. Taking the co-logarithm gives $\overline{1}.546$ as the answer. By the new method set the runner to .635 on scale LL0, read .454 on scale A, this is the co-logarithm which deducted from 0 gives $\overline{1}.546$ as the answer.

What is claimed is:

1. In a slide rule a logarithmic scale and a scale representing logarithms of co-logarithms of numbers less than unity, both of said scales being of the same suit length, whereby when the respective graduations on the scale representing logarithms of co-logarithms are in alignment with the graduations of the standard logarithmic scale representing the corresponding hyperbolic co-logarithms, the hyperbolic co-logarithms of numbers less than unity can be read directly from one to the other.

2. In a slide rule having relatively movable members, one logarithmic scale on one of said members, another identical logarithmic scale on another of said members and a scale representing logarithms of co-logarithms of numbers less than unity on one of said members, all three of said scales being to the same unit length whereby the hyperbolic co-logarithms of numbers less than unity can be multiplied or divided by any number of variables, the results being read as powers of said numbers on the scale representing logarithms of co-logarithms or as the hyperbolic logarithms thereof on the logarithmic scale on that member carrying the scale representing logarithms of co-logarithms.

ADOLF W. KEUFFEL.

CERTIFICATE OF CORRECTION.

Reissue No. 20,984.   January 24, 1939.

ADOLF W. KEUFFEL.

It is hereby certified that error appears in the printed specification of the above numbered patent requiring correction as follows: Page 2, second column, line 25, for "suit" read unit; and that the said Letters Patent should be read with this correction therein that the same may conform to the record of the case in the Patent Office.

Signed and sealed this 14th day of March, A. D. 1939.

Henry Van Arsdale.

(Seal)   Acting Commissioner of Patents.

answer. This operation involved a number of additional operations, both mentally and mechanically, and increased the liability of error, as the correctness of the result depended not on the
5 mechanical operation of the slide rule alone, but also upon the mental operation of the operator combined with the mechanical operation of the slide rule.

My slide rule has overcome this difficulty and
10 made it possible to handle roots and powers of numbers less than unity so as to obtain the solution of examples in formulae involving powers and roots of numbers less than unity directly on my slide rule by mere mechanical operation of the
15 same and without the use of reciprocals of the quantities handled, thereby eliminating any separate mental operation and the possibility of errors involved therein.

In the equation of the catenary curve and in
20 hyperbolic functions, the expression e^{-x} occurs in which $e=2.7182+$ is the base of natural or Napierian logarithms, also in many formulae the logarithms to the base e are required or is to be taken as a factor.
25 By placing the reciprocal of e on scale LL0 in alignment with an index "1" of the regular logarithmic scale A, the values of e^{-x} can be at once read for all values of x within the range of the scale, without the setting of the slide N and
30 also the co-logarithms to base e can be directly read on A.

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45 C read 3.05 on scale LL3, now keep 3.05 in mind and then set runner to this number 3.05 on C; and read the reciprocal .328 on CI which is the answer sought.

It will be noted that in this problem besides various operations of the slide and runner, it was necessary to take two readings mentally and reset the same on the slide rule, which may readily
5 cause an error.

By my new method all that is necessary to solve the above example is to set the left index of scale B to 0.8 on scale LL0 and opposite 5 on scale B read answer .328 on scale LL0.

Example 2
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Required $\log_e .635$.

Old method—set runner to .635 on C; read reciprocal 1.575 on CI, keep this number in mind and set runner to 1.575 on LL2; read .454 on D=$\log_e$
15 1.575. Taking the co-logarithm gives $\overline{1}.546$ as the answer. By the new method set the runner to .635 on scale LL0, read .454 on scale A, this is the co-logarithm which deducted from 0 gives $\overline{1}.546$ as the answer.
20
What is claimed is:

1. In a slide rule a logarithmic scale and a scale representing logarithms of co-logarithms of numbers less than unity, both of said scales
25 being of the same suit length, whereby when the respective graduations on the scale representing logarithms of co-logarithms are in alignment with the graduations of the standard logarithmic scale representing the corresponding hyperbolic co-
30 logarithms, the hyperbolic co-logarithms of numbers less than unity can be read directly from one to the other.

2. In a slide rule having relatively movable members, one logarithmic scale on one of said
35 members, another identical logarithmic scale on another of said members and a scale representing logarithms of co-logarithms of numbers less than unity on one of said members, all three of said scales being to the same unit length whereby the
40 hyperbolic co-logarithms of numbers less than unity can be multiplied or divided by any number of variables, the results being read as powers of said numbers on the scale representing logarithms of co-logarithms or as the hyperbolic loga-
45 rithms thereof on the logarithmic scale on that member carrying the scale representing logarithms of co-logarithms.

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