United States Patent

[11] 3,570,757

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[54] **SLIDE RULE FOR BELLEVILLE SPRINGS**
4 Claims, 11 Drawing Figs.

[52] **U.S. Cl.** **235/70**
[51] **Int. Cl.** **G06g 1/02**
[50] **Field of Search** 235/70, 70.1

[56] **References Cited**
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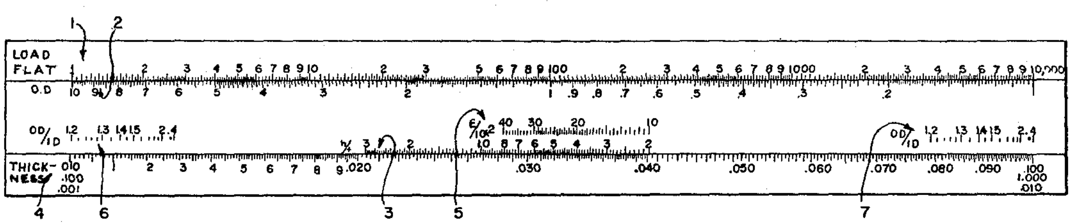
ABSTRACT: This invention provides a mechanically simple slide rule which will solve the following equations.

$$P=\frac{Ef}{(1-\sigma^2)\ Ma^2}[(h-f)\ (h-f/2)\ t+t^3] \qquad (1)$$

$$S_o=\frac{Ef}{(1-\sigma^2)\ Ma^2}[C_1\ (h-f/2)+C_2t] \qquad (2)$$

$$S_{t_2}=\frac{Ef}{(1-\sigma^2)\ a^2}[T_1\ (h-f/2)+T_2t] \qquad (3)$$



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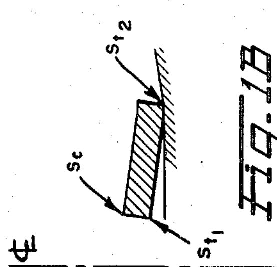

Fig. 1B

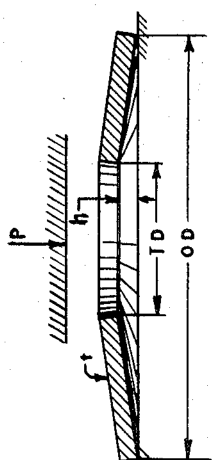

Fig. 1A

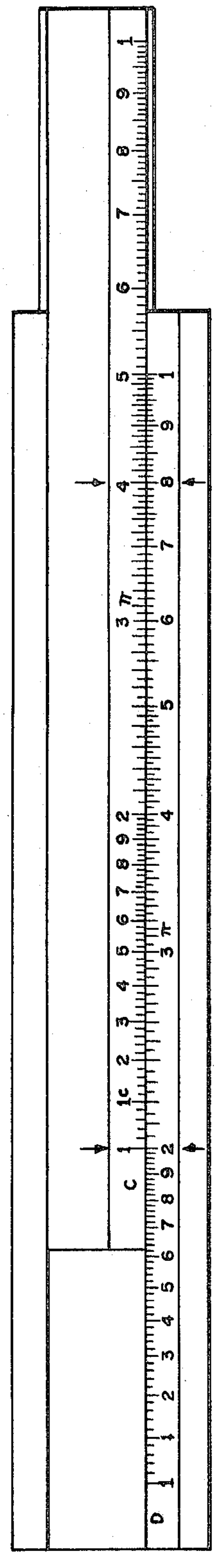

Fig. 2

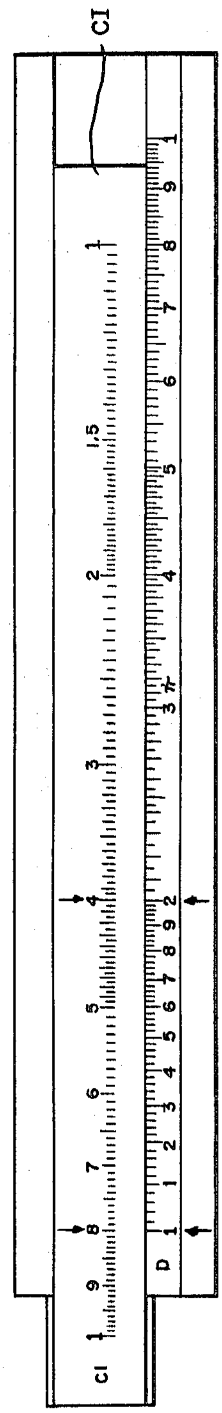

Fig. 3

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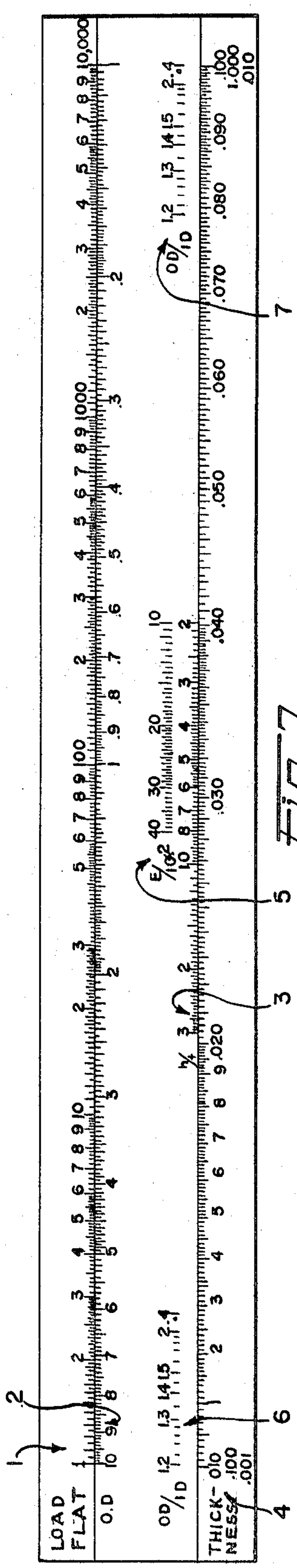

Fig. 7

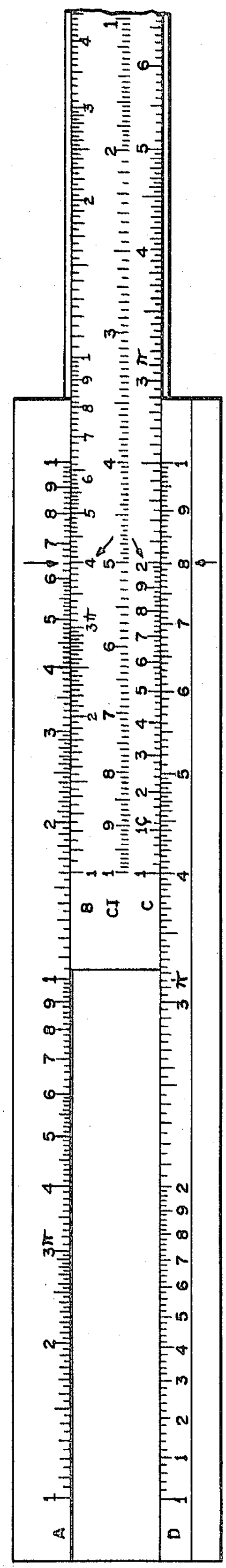

Fig. 5

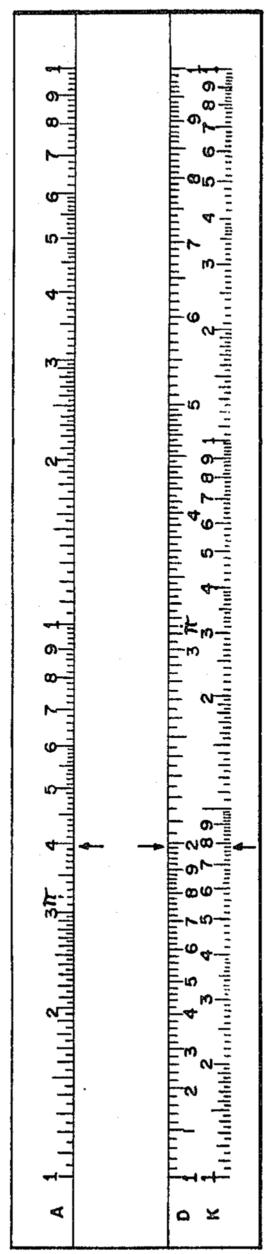

Fig. 4

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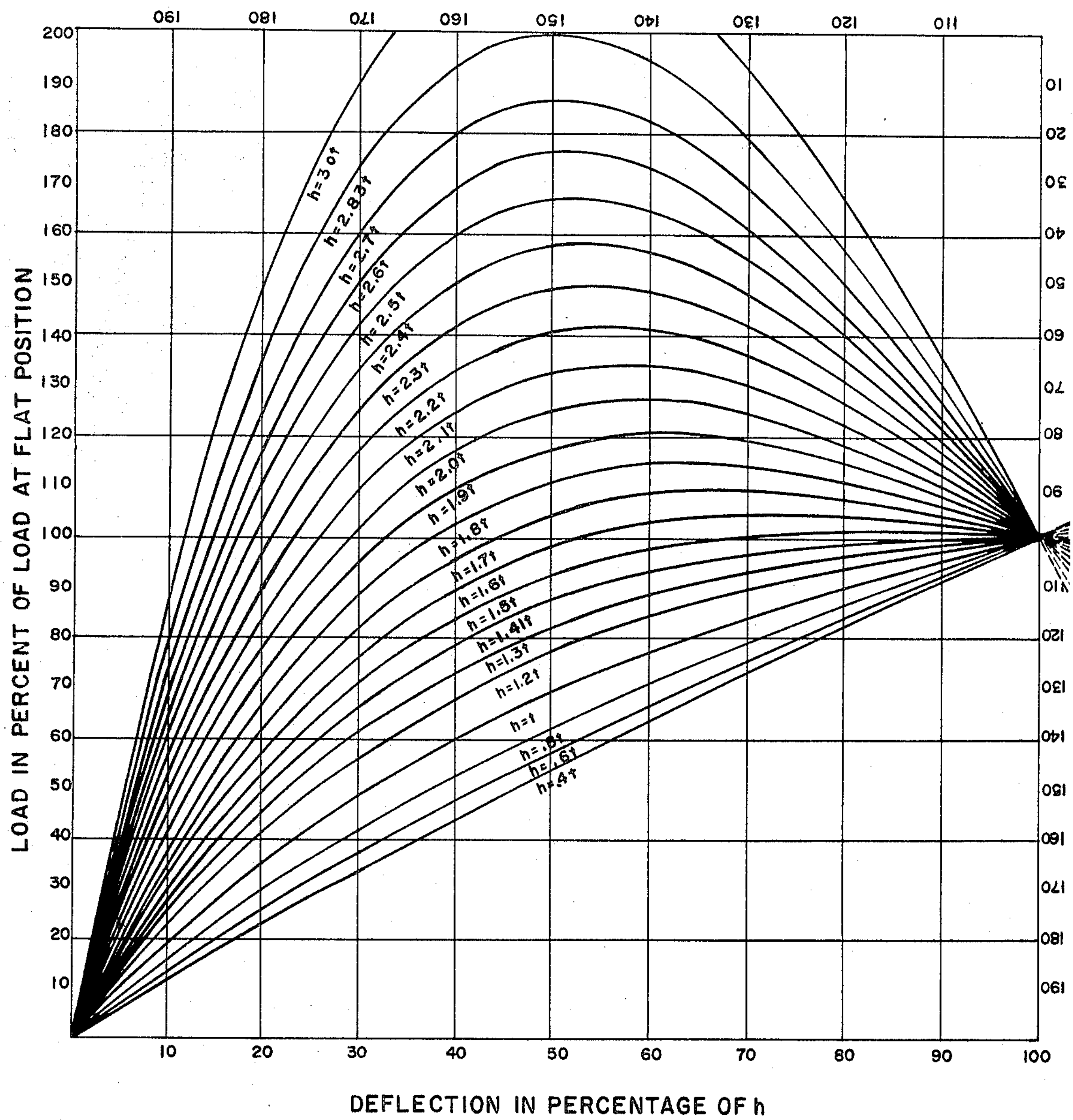

Fig. 6

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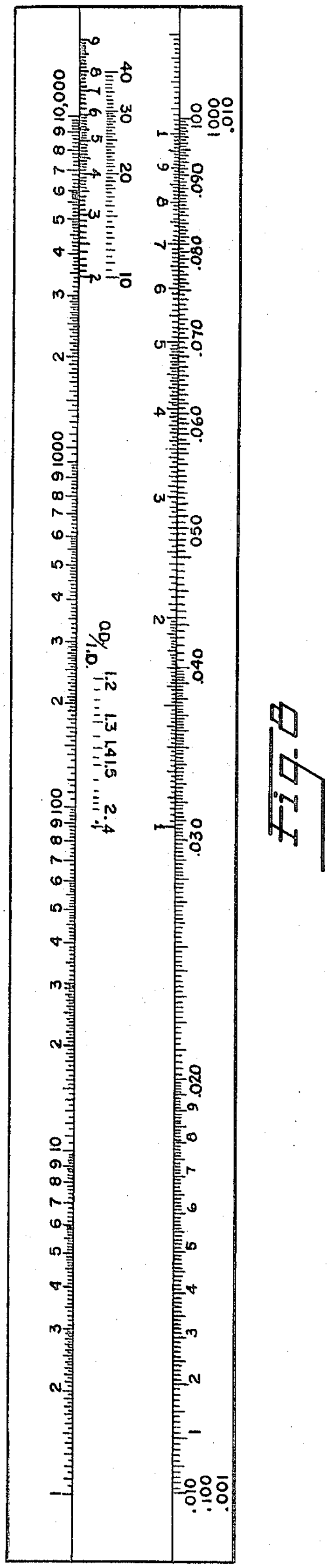

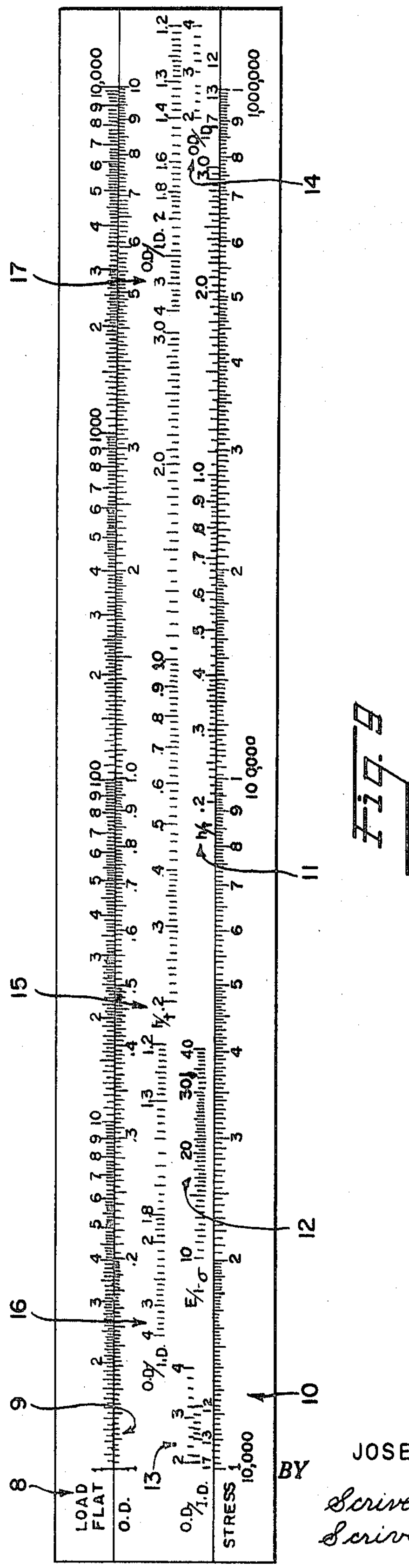

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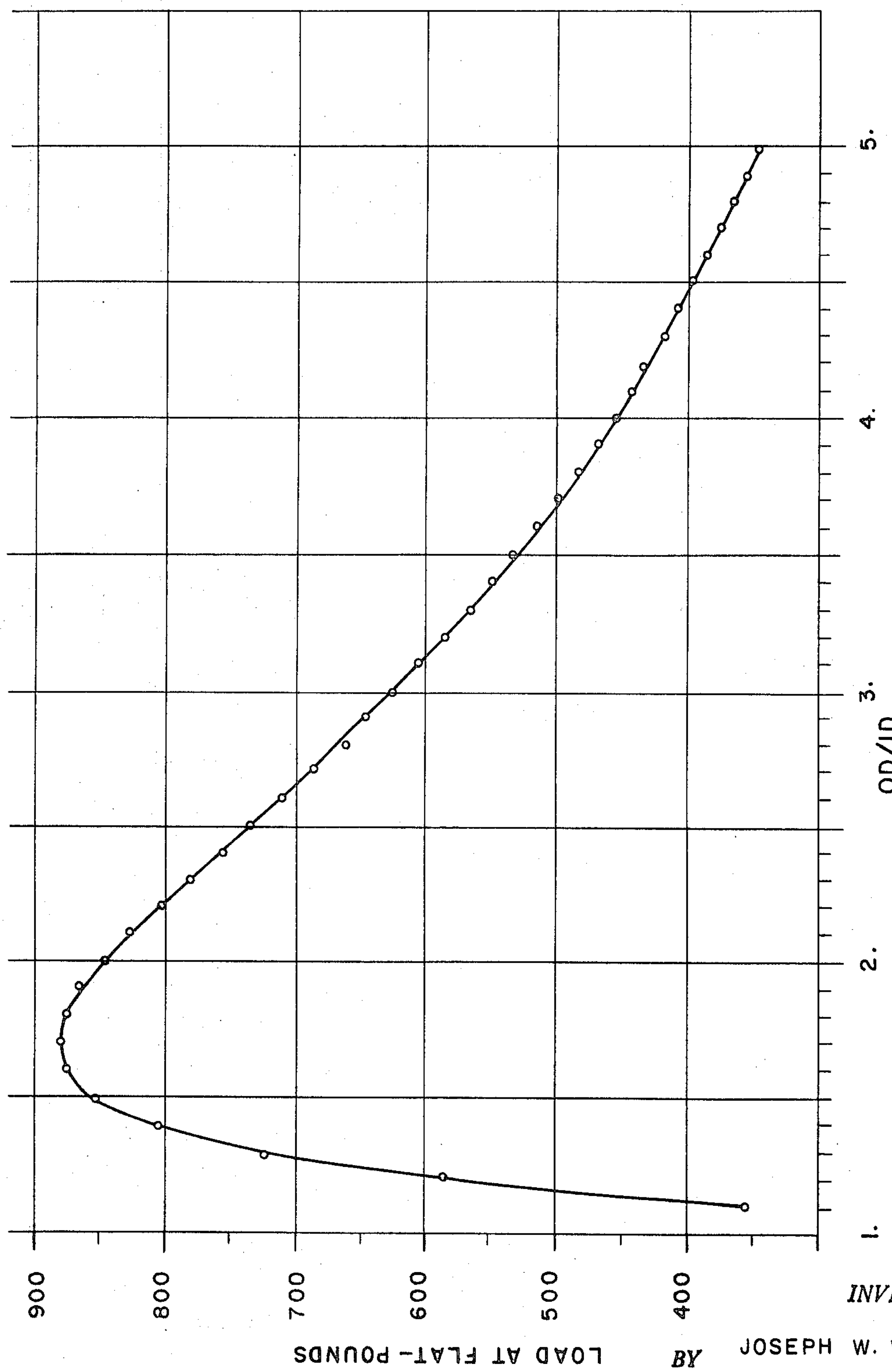

Fig. 10

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SLIDE RULE FOR BELLEVILLE SPRINGS

DESCRIPTION OF THE DRAWINGS

FIG. 1A is a diametrical sectional view of a Belleville spring washer;

FIG. 1B is a view of part of the washer shown in FIG. 1A;

FIG. 2 is an elevational view of part of a conventional standard slide rule;

FIG. 3 is an elevational view of the slide rule of FIG. 2 with an inverted scale;

FIG. 4 is an elevational view of part of a conventional slide rule in which each cycle of scale A is one-half as long as scale D, and each cycle of scale K is one-third as long as scale D;

FIG. 5 is an elevational view of a conventional slide rule having one slide and five scales, one of which is inverted;

FIG. 6 shows load-deflection curves for use with the computer provided by the invention, for determining intermediate loads when the load at flat is known;

FIG. 7 is an elevational view of the thickness side of the computer;

FIG. 8 is the same as FIG. 7 with the slide inverted;

FIG. 9 is an elevational view of the stress side of the computer; and

FIG. 10 is a graph showing variation in the load at flat with the *OD/ID* ratio.

DEFINITION OF TERMS

The following nomenclature is used throughout this specification, reference being made in part to the illustration of a Belleville washer in FIGS. 1A and 1B:

$R=OD/ID$

$a=OD/2$, in inches

c=percent of deflection to flat

$$C_1=\frac{6}{\pi 1nR}\left[\frac{R-1}{1n\ R}-1\right]$$

$$C_2=\frac{6}{\pi 1nR}\left[\frac{R-1}{2}\right]$$

E=Modulus of elasticity (30,000,000 for steel)

$$\frac{E}{1-\sigma^2}=\text{Material factor } (10^6 \text{ is omitted on the slide rule})$$

f=Deflection, in inches

h=Inside height, in inches

ID=Inside diameter, in inches

$$M=\frac{6}{\pi 1nR}\frac{(R+1)^2}{R^2}$$

OD=Outside diameter, in inches

P=Load, in lbs.

P_F Load at flat position

$q=h/t$

S_c Compressive stress on convex side of inner edge, in p.s.i.

S_t Tensile stress at concave side of inside diameter, in p.s.i.

S_t Tensile stress at concave side of outer edge, in p.s.i.

σ Poisson's ratio

t=Thickness, in inches

$$T_1=\frac{R\ 1n\ R-(R-1)}{1n\ R}\times\frac{R}{(R-1)^2}$$

$$T_2=\frac{0.5R}{R-1}$$

THE PRIOR ART

The derivations of formulas and the logic of the invention will be clarified by an explanation of a simple slide rule. FIG. 2 illustrates a conventional rule having two fixed side parts and one slide. Any two adjacent scales on the rule and the slide provide for the solution of the problem $xy=z$. This is accomplished by the graphic addition of the logarithms or:

$$\log_{10} x+\log_{10} y=\log_{10} z.$$

FIG. 2 illustrates a standard slide rule which is adjusted so that $x=2$, $y=4$ and $z=8$ and solves the equation $2\times4=8$. This also illustrates division by the graphic subtraction of logarithms. In FIG. 2, 84x

If one of the scales is inverted so that values increase from right to left the same operations may be performed by dealing with the reciprocal of one of the factors, e.g., $2\times4=\frac{2}{\frac{1}{4}}$; the equation 2—xbeing illustrated by scale *CI* in FIG. 3. Powers and roots of numbers are handled by varying the proportions of the scales. In FIG. 4 each cycle of scale A is half as long as scale D and each cycle of scale K is one-third as long as scale D, and the setting illustrates the equations 2^2x4 and 2^3x 8. In a conventional slide rule scales are aligned with each other, but in a special slide rule one of the scales is offset from the others, this being equivalent to introducing a constant into an equation. All settings of the variables will also include the constant. Remembering the objective of keeping the computer simple, there may be only one slide and, therefore, only four variables, as illustrated by scales A, B, C and D in FIG. 5. However, any of the variables may be raised to a power and a constant may be present in this simple construction. The basic relation of the scales is $\frac{A}{B}=\frac{D}{C}$

FIG. 5 illustrates this with the following values:

$$A=64, B=4, C=2^2, D=8^2$$

$$\frac{64}{4}=\frac{8^2}{2^2}$$

DESCRIPTION OF THE INVENTION

All of the foregoing is common practice in the design of standard and special slide rules. Others have coped with complicated formulas by adding extra slides, rotating arms and variable scales, but these devices are expensive to build and require special training in their use. In contrast, my invention consists in the ingenious manipulation of the formulas so that complicated formulas may be solved on a simple slide rule with the aid of one chart. One of the principal objects of the invention is to focus the designers' attention on the ratio of height to thickness which determines the shape of the load-deflection curves as shown in FIG. 6. Others have introduced arbitrary factors to aid in the solution of the formulas (see for example, "Society of Automotive Engineers Handbook Supplement," HS 63). Graphic representations of these factors conceal the basic shape of the load-deflection curves because they include elements of *OD/ID* and *f/t*. In my invention *h/t* is a principal factor in the transpositions of all three basic equations. In describing the computer we will derive the mathematical equations first and then explain the placement of the scales on the computer.

The first step in the simplification of equation (1) is due to the observation that the formula for load at flat position (when $f=h$) eliminates $(h-f)$, $(h-f/2)$ at t, resulting in the following equation:

$$P_F=\frac{Eht^3}{(1-\sigma^2)Ma^2} \qquad (4)$$

The next step is the observation that loads at intermediate deflections depend upon the ratio of height to thickness as shown by the following: Divide equation (1) by equation (4). This gives percent of intermediate load to load at flat.

$$\frac{P}{P_F}=\frac{f}{ht^3}\left[(h-f)\left(h-\frac{f}{2}\right)t+t^3\right]$$

Now let c=percent of deflection to flat=f/t. Then $f=ch$ and

$$\frac{P}{P_F}=\frac{ch}{ht^3}\left[(h-ch)\left(h-\frac{ch}{2}\right)t+t^3\right]$$

$$=\left(\frac{h}{t}\right)^2 c(1-c)\left(1-\frac{c}{2}\right)+c \qquad (5)$$

The chart shown in FIG. **6** was constructed from this equation, and with this chart it is a simple matter to determine any intermediate load when the load at flat P_F is known. The solution of equation (4) on the slide rule is, therefore, adequate to solve equation (1) with the aid of the chart of FIG. **6**.

Equation (4) contains six variables:

P_F — load factor
$E/1\text{-}\sigma^2$ — **material factor**
h — height factor
t^3 — thickness factor
M — ratio of OD/ID factor
a^2 — outside diameter factor

I reduce this to four variables and introduce wanted factors as follows:

$$q=\frac{h}{t}\quad a^2=\frac{\overline{OD}^2}{4}$$

$$P_F=\frac{4Eqt^4}{(1-\sigma^2)M\,\overline{OD}^2}$$

$$\text{let } \alpha=\frac{4E}{1-\sigma^2M} \text{ then } P_F=\alpha\frac{qt^4}{\overline{OD}^2} \qquad (6)$$

This is an important element of invention because it introduces the ratio of height to thickness which relates to FIG. **6**.

By substituting steel values for the material factors and (OD/ID=2a becomes a constant and equation (6) has the four variables and a constant required to fit a simple slide rule.

σ=0.3
E=3.0×10^7
M=0.685
α=1.93×10^8

Refer now to FIG. **7** which is the thickness side of the computer. Scale **1** is P_F in equation (6), scale **2** is $\overline{OD}^2$ in equation (6), scale **3** is q in equation (6) and scale **4** is t^4 in equation (6). Scales **1**, **2**, and **4** are aligned according to their indices. Scale **3** is located by solving formula 6 for any assumed values.

For example: $P_F=100$, $OD=1$, $q=1$

$$t=\sqrt[4]{\frac{P_F\overline{OD}^2}{q}}=.0268$$

The length of one cycle of each scale is determined by the exponent attached to that factor in equation (6). P_F and q having the same exponent, one, scales **1** and **3** have the same base length. $\overline{OD}^2$ having the exponent 2, each cycle of scale **2** is twice as long and t having the exponent 4, each cycle of scale **4** is four times the base scale length. This principle is illustrated in FIG. **4**. The inverse direction of scales **2** and **3** will be discussed later.

Referring to equation (4,it is apparent that P_F for any other material than steel would be proportional to the revised value of the material factor $\frac{E}{1-\sigma^2}$

Scale **5** is printed on the slide to make this correction when necessary. Since the exponent is 1, the length of one cycle is the same as scale **1**. The scale may be located anywhere on the slide. The direction will be discussed later.

Referring again to equation (4), it is apparent that P_F is proportional to the OD/ID factor $1/M$. However, $1/M$ is itself a complex equation involving natural logarithms, whereas the scales on the slide rule are common logarithms. The value of M assumed in order to locate scale **3** was 0.685 for a ratio OD/ID=2. Other typical values, M', for other ratios of OD/ID and the corresponding correction factors are as follows:

OD/ID	M'	M/M'
1.2	0.291	2.35
1.3	0.388	1.76
1.5	0.523	1.31
4.	0.775	0.89

The adjusting scale for this factor is marked **6** in FIG. **7**. Since the factor M has the same exponent as the P_F scale, the intervals are the same. Scale **6** is inverted for a reason which will be discussed later, but the logic of its construction may be seen in FIG. **8** in which the slide has been inverted; this shows the agreement of the scale with the above table. Scale **7** is identical to scale **6**. It is repeated so that one scale or the other will be visible under the cursor regardless of the position of the slide.

STRESS SIDE

Equation (2) is a stress-deflection formula. To make it compatible with the load-thickness side, it is necessary to combine equations (1and (2in order to relate load at flat to stress at flat. This provides the common denominator of load at flat between the two sides of the rule.

It is common practice in spring design technique to combine load deflection and load stress formulas. This is usually done by dividing or multiplying one formula by the other. This is not applicable to equations (1and (2) because dividing would eliminate too many of the variables and multiplying would produce an exceedingly complex equation. In accordance with the invention I combine the formulas by substitution, retaining the ratio of height to thickness and other required variables in a relatively simple form.

By substitution of qt for h equation (4) may be written:

$$t^2=\sqrt{\frac{P_F(1-\sigma^2)Ma^2}{Eq}} \qquad (7)$$

For stress at flat position equation (2) may be written:

$$S_{CF}=\frac{Eqt^2}{(1-\sigma^2)Ma^2}\left(\frac{C_1q}{2}+C_2\right) \qquad (8)$$

Now combine (7) and (8) by substitution:

$$S_{CF}=\frac{Eq}{(1-\sigma^2)Ma^2}\sqrt{\frac{P_F(1-\sigma^2)Ma^2}{Eq}}\left(\frac{C_1q}{2}+C_2\right)$$

$$\text{simplify}\quad S_{CF}=\frac{\sqrt{P_F}\,\sqrt{q}}{OD}\sqrt{\frac{E}{(1-\sigma^2)M}}\,(C_1q+2C_2)$$

$$\text{Let } \beta=\sqrt{\frac{E}{1-\sigma^2}}\sqrt{\frac{1}{M}}\,(C_1q+2C_2)\,\sqrt{q} \qquad (9)$$

$$\text{then } \frac{\sqrt{P_F}}{OD}=\frac{S_{cF}}{\beta} \qquad (10)$$

This is the form of the basic slide rule $\frac{A}{B}=\frac{D}{C}$ as illustrated in FIG. **5**, except for the fact that q, the factor for h/t cannot be isolated from C_1 and C_2 which depends upon OD/ID. The scale for q, therefore, cannot be a simple logarithmic scale. This would normally be considered an insurmountable obstacle to the construction of a simple slide rule.

I solve this problem by substituting basic values in equation (10) as follows.

E=10×10^6

$\sigma=0.3$
$C_1=1.22$
$C_2=1.378$
$M=0.689$
$OD=1$
$PF=100$
which gives

$$S_{CF}=6.918\times10^4\sqrt{q}\,(1.22q+) \qquad (11)$$

This equation was solved by computer for the 91 values of q. The placement of this scale may be verified by substituting a value of 1 for q in equation (11). Then $S_{CF}=275{,}000$.

Referring now to FIG. 9, scales **8**, **9** and **10** are located and spaced in accordance with principles previously discussed. Scale **11** is spaced according to equation (11) and located with $S_{CF}=275{,}000$ aligned with $h/t=1$ as shown in the above example.

Scale **12** is the same as scale **5** on the thickness side.

Scale **13** was established in the same way as scale **6** on the thickness side, but it is much more complex because equation (9) contains C_1 and C_2 as well as M. The complete equation and a graph of the function are shown in FIG. **10**. This scale is labeled below the line for increasing values of P_F as OD/ID varies from 1.2 to 1.7 and above the line for decreasing values as OD/ID varies from 1.7 to 4.0. 1.7 is the peak of the curve in FIG. **10**.

Scale **14** is the same as scale **13**.

Equation (3) is the formula for stress at the point S_t in FIG. 1B. Scales **15**, **16** and **17** correspond for this equation to scales **11**, **13** and **14** for equation (2) and they were derived, spaced and located in a similar manner.

DIRECTION OF SCALES

For ease of use it is desirable to locate similar scales in the same sequence on both sides of the rule. This I do as follows.

	Thickness Side	Stress Side
Standard Scale (Figure 5):		
A	Load flat	Load flat.
B	OD	OD.
C	h/t	h/t.
D	Thickness	Stress.

On the stress side of the rule, equation (10) is in the form

$$\frac{A}{B}=\frac{D}{C}$$

and, therefore, the scales read from right to left the same as FIG. **5**. On the thickness side the equation (6) is in the form $AB=CD$. I put this in the desired form by writing equation (6)

$$\frac{P_F}{\frac{1}{OD^2}}=\frac{t^4}{\frac{1}{q}}$$

This requires that scales **2** and **3** be inverted and multiplication is accomplished by division by the reciprocal as illustrated in FIG. 3.

Each of the auxiliary scales, **5**, **6**, **7**, **12**, **13**, **14**, **16** and **17** has an arrow to indicate the standard value on which the rule is based without adjustment. For example, scale **5** has an arrow at **33** which is the material factor for steel. An adjustment may be made either by (a) setting the hairline over the arrow and moving the slide to the new value or (b) setting the hairline over the new value and moving the slide to the arrow. The direction of the scales has been established so that procedure (a) will be used when progressing from the lower to the upper main scales and procedure (b) will be used when progressing from the upper main scales to the lower.

SPECIAL NOTATIONS

On scale **6** there is a dot between the digits **2** and **4** which represents 2.5. This notation is used because the function is changing very rapidly at this point. Standard notation would call for a line at **3**, but this would be practically superimposed on **4**.

On the stress side of the slide rule there is a red circle which overlaps scales **8** and **9** on the extreme right. This is to remind the operator to replace the slide so that it faces correctly.

I claim:

1. A slide rule for use in designing a Belleville spring, comprising two spaced elongated fixed members, a slide between said members and coextensive therewith, and a cursor mounted on the fixed members for movement along them, the parts of the rule having two sides on which there are the following scales:

a. On one side of the rule scales for determining thickness in inches according to the formula:

$$P_F=\alpha\frac{qt^4}{OD^2}$$

in which
P_F=load at flat position

$$\alpha=\frac{4E}{1-\sigma 2M}$$

E=modulus of elasticity (30,000,000 for steel)
σ=Poisson's ratio

$$M=\frac{6}{\pi LnR}\,\frac{(R-1)^2}{R^2}$$

$q=h/t$
h=inside height, in inches
t=thickness
OD=outside diameter, in inches,
in which formula steel values are substituted for material values and $OD/ID=2$ so that α becomes a constant and
$\sigma=0.3$
$E=3.0\times10^7$
$M=0.685$
$\alpha=1.93\times108$
one of these scales representing values of P_F from 1 to 10,000 extending from end to end of one of the fixed members, a second scale representing values of OD^2 from 10 down to 0.1 on the slide adjacent and coterminous with the P_F scale, a third scale representing values of t from 0.001 to 1.0 inch on the second fixed member and coterminous with P_F and OD^2 scales, a fourth scale on the slide representing values of q from 1.0 to 3 in the direction of 0.010 on the t scale and from 1.0 to 2 in the opposite direction and with the 1.0 value of q aligned with 0.0268 on the t scale if it is assumed that $P_F=100$, $OD=1$ and $q=1$, and at least one scale on the slide representing values of OD/ID from 1.2 to 4,

b. and on the other side of the rule scales for determining the stress at flat position according to the following formula:

$$\sqrt{\frac{P_F}{OD}}=\frac{ScF}{\beta}$$

in which
P_F=load at flat position
OD=outside diameter
ScF=compressive stress on convex side of the inner edge of flat, in p.s.i.

$$\beta=\sqrt{\frac{E}{1-\sigma^2}}\sqrt{\frac{1}{M}}\,(C_1q+2C_2)\sqrt{q}$$

E=modulus of elasticity (30,000,000) for steel
σ=Poisson's ratio

$$M=\left[\frac{6}{\pi LnR}\right]\left[\frac{(R-1)^2}{R^2}\right]$$

$$C_1=\left[\frac{6}{\pi LnR}\right]\left[\frac{R-1-1}{LnR}\right]$$

$q=h/t$

$$C_2=\left[\frac{6}{\pi LnR}\right]\left[\frac{R-1}{2}\right]$$

$R=OD/ID$

one of these scales representing values of P_F from 1 to 10,000 and extending from end to end of one of the fixed members, a second scale representing values of OD from 0.1 to 10 on the slide adjacent and coterminous with the P_F scale, a third scale representing values of ScF from 10,000 to 1,000,000 on the second fixed member and coterminous with the P_F scale, a fourth scale on the slide adjacent the ScF scale representing values of h/t from 1.0 down to 0.2 in the direction of the lower end of the ScF scale and from 1.0 to 3.0 in the other direction, with 1.0 on the h/t scale at 275,000 on the ScF scale when the ScF and P_F scales are coterminous and when it is assumed that the terms of the β equation have the following values:

$E=10\times 10^6$
$\sigma=0.3$
$C_1=1.22$
$C_2=1.378$
$M=0.689$
$OD=1$
$P_F=100$

2. A slide rule according to claim 1, comprising in addition at least one scale on the stress side of the slide representing values of OD/ID from 1.2 to 4, for use in determining stress OD/ID is other than 2.

3. A slide rule according to claim 1, comprising in addition a scale on each side of the slide representing values of $E/(1-{}^2)$ from 10,000,000 to 40,000,000 for use when E is other than 30,000,000 and σ is other than 0.3.

4. A slide rule according to claim 1, comprising in addition at least one scale on the thickness side of the slide representing values of OD/ID from 1.2 to 4.0 for use in determining thickness when OD/ID is other than 2.