## INSTRIUCTIONS

## for the Use of

MODEL F-6B6
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```
    Rec. to Folar
Opsite the larger no. on P set the left index
of Q. Move hair line to smeller no. on Q read
absolute value on P. Move hair line to smaller
no. on D. On C set larger No. under hair Iine
Move Hair line to riçht index of C and read
Radan of Angle on TR-1. If right index is off
scale use left index and read mR-2.
        Convert Radan to Degree
```


## INSTRUCTIONS

## FOR THE USE OF F-686 SLIDE RULE

## 1. GENERAL DESCRIPTION

This slide rule has been designed for expert electrical engineer to simplify the vatious calculations occurred frequently in electricity, namely not only the computation of multiplication and division can be done with the A, B, C, D, $\mathrm{CI}, \mathrm{DF}$ and $C F$ scales, but also the $\mathrm{LL}_{8}, L L_{2}, L L_{1}{ }^{\prime}, L L_{08}, L_{02}$ and $L L_{02}$ scales make it possible to obtain the result of $a^{n}$, $e^{x}$ and $\log _{c} N$. Moreover, the $P, Q$ and $P^{\prime}$ scales are essential for vector computation, the $\mathrm{Sh}_{2}, \mathrm{Sh}_{1}$ and Th scales for hyper bolic function.
2. SCALE ARRANGEMENT AND USAGE
(Front Face) $\mathrm{Tr}_{1}, \mathrm{Tr}_{2}, \mathrm{P}^{\prime}, \mathrm{P}, \mathrm{Q}, \mathrm{ST}, \mathrm{Sr}, \mathrm{S} \theta, \mathrm{C}, \mathrm{D}, \mathrm{LL} \mathrm{L}_{\mathrm{os}}$, $\mathrm{LL}_{\mathrm{ca}}, \mathrm{LL}_{01}$
(Back Face) $\mathrm{Sh}_{1}, \mathrm{Sh}_{2}, \mathrm{DF}, \mathrm{A}, \mathrm{B}, \mathrm{CF}, \mathrm{Th}, \mathrm{Cl}, \mathrm{C}, \mathrm{D}, \mathrm{LL}$, $\mathrm{IL}_{2}, \mathrm{db}, \mathrm{LL}_{2}{ }^{\prime}$.
(a) $T_{1}{ }_{1}$ and $T_{r_{2}}$

These scales are used in the computation of the tangent of an angle from 0.1 to 0.8 radians, and from 0.8 to 1.472 radians respectively.
(b) P, Q and P'

The computation of the vector can be conveniently done by these scales as ordinary multiplication and division done
by the $C$ and $D$ scales.
(c) ST

This scale gives the sine and tangent of an angle less than $6^{\circ}$.
(d) Sr and $\mathrm{S} \theta$

These scales are used to obtain the sine and cosine of an angle, co-operate with the $P$ and $Q$ scales. Angles are graduated at degree and its decimal fraction in the Sr scale, and at radian in the S8 scale. Thus conversion from degree to radian or its reverse process is made by the use of these scales.
(e) LLos, LLoz and $L_{01}$

These are called the reciprocal Log Log scales and are used to find the values of the type form of $a^{-n}, e^{-x}$ and $\log _{e} N$.
(f) Sh1 and Sha

These scales make it possible to compute the sine-hyperbolic function, which is frequently necessary in alternating current theory.
(g) Th

This gives the value of the tangent-hyperbolic function. (h) $\mathrm{LL}, \mathrm{LL}_{2}$ and $\mathrm{LL}_{1}^{\prime}$

The $L L_{,}$and $L L I_{1}$ are called the Log Log scales and are used to find the values of the type form of $a^{n}, e^{x}$ and $\log _{e} N$.
$L_{L_{1}}{ }^{\prime}$, which as an extension scale of C , is marked in red at the right end of it, substitutes the ordinary ${L L_{2}}^{\text {scale. }}$
(i) db

This decibel scale is useful in the computation of electric communication circuit. Moreover this equaly subdivided scale can be used as ordinary mantissa scale to obtain the com mon logarithms of a number.
(i) $A, B, C, D, C I, D F$ and $C F$

Of these scales, the DF and CF scales are the $D$ and $C$ scales folded at $\pi$, so every number on these scales is equal to the number on the D and C scales multiplied by $\pi$.

## 3. TRIGONOMETRIC FUNCTIONS

(1) $\operatorname{Sin} \theta$

The sine of an angle, either in degree or in radian can be directly read on $\mathrm{S} \theta$ or Sr scale.

Example 1. $\sin 51.8^{\circ}=0.786$
Move hairline to 51.8 on $\mathrm{S} \theta$,
under hairline find 7.86 on Q ,
read answer as 0.786 .
Example 2. $\sin 0.665=0.617$
Move hairline to 0.665 on Sr ,
under hairline find 6.17 on $Q$,
read answer as 0.617.
The value of the sine or tangent of an angle less than $6^{\circ}$ is calculated by the use of the ST scale.

Example 3. $\sin 1.5^{\circ}=0.0262$
Move hairline to 1.5 on ST,
under hairline find 2.62 on $C$,
read answer as 0.0262 .
(2) $\operatorname{Cos} \theta$

Example 4. $\cos 51.8^{\circ}=0.618$
Move hairline to tight index of $P$,
set 51.8 on $\mathrm{S} \theta$ under hairline,
opposite left index of $Q$ find 6.18 on $P$,
read answer as 0.618 .
Example 5. $\cos 0.665=0.787$

Move hairline to right index of $P$, set 0.665 on Sr under hairline, opposite left index of $Q$ find 7.87 on $P$, read answer as 0.787 .
(3) Conversion between Degree and Radian

Conversion from degree to radian or its reverse process is made by using the Sr and $\mathrm{S} \theta$ scales.

Example 6. $53^{\circ}=0.925$ radians
Move hairline to 53 on $\mathrm{S} \theta$,
under hairline read answer as 0.925 on Sr .
Example 7. 0.52 radians $=29.8^{\circ}$
Move hairline to 0.52 on Sr ,
under hairline read answer as 29.8 on $\mathrm{S} \theta$.
In the case of conversion of a small angle, move the decimal point one place to the right, perform the oparation as is explained above, and read the answer moving the decimal point one place lower.

Example 8. 0.06 radians $=3.44^{\circ}$
Move hairline to 0.6 on Sr ,
under hairline find 34.4 on $S \theta$,
read answer as 3.44
(4) $\operatorname{Tan} \theta$

The $\mathrm{Tr}_{1}$ and $\mathrm{Tr}_{2}$ scales represent a single scale of angles ranging from 0.1 to 1.472 radians.

When using the $\mathrm{Tr}_{1}$ scale to read the value of $\tan \theta$, read the left index of $C$ as 0.1 and the right index as 1 , and using the $\mathrm{Tr}_{2}$ scale to read the value of $\tan \theta$, read the left index of $C$ as 1 and the right index as 10 .

Example 9. $\tan 0.35=0.365$
Move hairline to 0.35 on $\mathrm{Tr}_{1}$,
under hairline find 3.65 on D ,
read answer as 0.365 .
Example 10. $\tan 1.17=2.36$
Move hairline to 1.17 on $\mathrm{Tr}_{1}$,
under hairline read answer as 2.36 .

## 4. VECTOR PROBLEMS

(1) The Absolute Value of Vector

The absolute value of vector, represented in the type form of $a+j b$, is equal to $\sqrt{a^{2}+b^{2}}$, and by the use of the $P, Q$ and $P^{\prime}$ scales this value can be computed very easily in the same operation as ordinary multiplication and division. Namely set zero on the $Q$ scale to $a$ on the $P$ scale, opposite $b$ on the $Q$ scale read $\sqrt{a^{2}+b^{2}}$ on the $P$ scale. When $b$ on the $Q$ scale runs off the $P$ scale, then set 10 on the $Q$ scale to $a$ on the $P$ scale, and opposite $b$ on the $Q$ scale read the answer on the $P^{\prime}$ scale.

Example 11. Find the absolute value of $3+j 4$.
Opposite 3 on $P$, set left index of Q ,
move hairline to 4 on $Q$,
under hairline read answer as on $P$.
Example 12. $\sqrt{8.76^{2}+9.32^{2}}=12.79$
Opposite 8.76 on $P$, set right index of $Q$, move hairline to 9.32 on Q , under hairline read answer as 12.79 on $\mathrm{P}^{\prime}$.
The computation of the type form of $\sqrt{a^{2}-b^{2}}$ an be done in the same way.

Example 13. $\sqrt{10.53^{2}-5.96^{2}}=8.68$
Move hairline to 10.53 on $\mathrm{P}^{\prime}$,
set 5.96 on $Q$ under hairline,
opposite right index of $Q$, read answer as 8.68 on $P$.
Example 14. $\sqrt{93.4^{2}-41.8^{2}}=83.5$
Calculate as $10 \times \sqrt{9} 34^{2}-4.18^{2}$
move bairline to 9.34 on $P$,
set 4.18 on $Q$ under hairline,
opposite left index of $Q$, find 8.35 on $P$,
read answer as $10 \times 8.35=83.5$.
(2) Phase Angle of Vector

In the vector of the type form of $a+j b$, the phase angle $\theta$ is represented as follows:

$$
\theta=\tan ^{-1} \frac{b}{a}
$$

Example 15. $\tan ^{-1} \frac{3.6}{2.5}=0.964$
Move hairline to 3.6 on D,
set 25 on $C$ under hairline, move hairline to left index of $C$, under hairline read 0.964 on $\mathrm{Tr}_{2}$.
Example 16. $\tan ^{-1} \frac{2.5}{3.6}=0.607$
Move hairline to 2.5 on D,
set 3.6 on $C$ under hairline, move hairline to right index of C , under hairline read 0.607 on $\mathrm{Tr}_{1}$.
(3) Conversion of Coordinate System

From the following relations;

$$
\begin{aligned}
a+\mathrm{j} b=\sqrt{a^{2}+b^{2}} / \tan ^{-1} \frac{b}{a} & =R / \theta \\
\text { here } \quad R & =\sqrt{a^{2}+b^{2}} \\
\theta & =\tan ^{-1} \frac{b}{a}
\end{aligned}
$$

$$
R \prime \theta=R \cdot \cos \theta+j R \cdot \sin \theta
$$

the conversion of vector in polar coordinate system to rectangular coordinate system and its reverse computation can be easily done.

Example 17. $-7.5+j 6.0=9.6 / \pi-0.675$
This is the example of conversion of $a+j b$ to polar coordinate system.

Opposite 7.5 on $P$, set left index of $Q$,
move hairline to 6.0 on Q ,
under hairline read absolute value as 9.6 on P .
Move hairline to 6.0 on D ,
set 7.5 on $C$ under hairline,
move hairline to right index of C ,
under hairline read 0.675 on $\mathrm{Tr}_{1}$,
thus the phase angle is equal to $\pi-0.675$.
Example 18. $25 / 52^{\circ}=15.4+j 19.7$
This is the example of conversion of polar coordinate system to $a+j b$.

Move hairline to right index of P , set 52 on $\mathrm{S} \theta$ under hairline, under hairline read $\sin 52^{\circ}=0.788$ on $Q$,
opposite left index of $Q$, read $\cos 52^{\circ}=0.616$ on $P$,
thus, real part is calculated as $25 \times 0.616=15.8$ and imaginary part $j 25 \times 0.788=j 19.7$.
(4) Multiplication and Division of Vectors

Multiplication and Division of two vectors have been computed from the following formulas:

$$
\begin{aligned}
& R_{1}\left|\theta_{1} \times R_{2}\right| \theta_{2}=R_{1} \cdot R_{1} / \theta_{2}+\theta_{2} \\
& R_{1} \left\lvert\, \theta_{1}=\frac{R_{1}}{R_{2}} / \theta_{1}-\theta_{3}\right. \\
& R_{2} / \underline{\theta_{2}}
\end{aligned}
$$

Example 19. $\frac { 3 - j 2 } { 8 + j 4 } = 0 . 4 0 4 \longdiv { 1 . 0 5 2 }$

$$
\begin{aligned}
\frac{3-j 2}{8+j 4} & =\frac{\sqrt{3^{2}+2^{2}} / \tan ^{-1}\left(\frac{-2}{3}\right)}{\sqrt{8^{2}+4^{2}} / \tan ^{-1}\left(\frac{4}{8}\right)}=\frac{3.61 /-0.588}{8.94 / 0.464} \\
& =0.404 /-0.588-0.464 \\
& =0.404 /-1.052=0.404
\end{aligned}
$$

## 5. HYPERBOLIC FUNCTIONS

The logarithmic scales $\mathrm{Sh}_{1}, \mathrm{Sh}_{2}$ and Th are used for the computation of hyperbolic function.
(1) $\sinh x$

The $S h_{1}$ and $S h_{1}$ scales give the value of $\sinh x$. When using the $\mathrm{Sh}_{1}$ scale to read the value of $\sinh x$, read the left index of $D$ as 0.1 and the right index as 1 , and using the $S h_{x}$ scale to read the value of $\sinh x$, read the left index of $D$ as 1 and the right index as 10 .

Example 20. sint $0.362=0.370$
Move hairline to 0.362 on $\mathrm{Sh}_{2}$,
under hairline find 3.70 on $D$,
read answer as 0.370 .
Example 21. sinh $2.56=6.42$
Move bairline to 2.56 on $\mathrm{Sh}_{2}$, under hairline read answer as 6.42 on D .
(2) Tanh $x$

The $\operatorname{Th}$ scale gives the value of $\operatorname{anh} x$. When using the $T$ scale to read the value of $\tanh x$, read the left index of $C$ as 0.1 and the right index as 1.

Example 22 $\tanh 0.183=0.181$
Move hairline to 0.183 on Th ,
under hairline find 1.81 on $C$, read answer as 0.181.
(3) $\operatorname{Cosh} x$

The value of $\cosh x$ can be computed from the following formula:

$$
\cosh x=\frac{\sinh x}{\tanh x}
$$

Example 23. $\cosh 0.575=1.170$
Move hairline to 0.575 on $\mathrm{Sh}_{1}$,
set 0.575 on Th under hairline,
opposite left index of $C$ read answer as 1.170 on $D$.

## 6. HYPERBOLIC FUNCTIONS OF COMPLEX ANGLES

Hyperbolic functions of complex angles are given from the following formulas:
(a) $\sinh (a+j b)=\sinh a \cdot \cos b+j \cosh a \cdot \sin b$

$$
=\sqrt{\sinh ^{2} a+\sin ^{1} b} / \tan ^{-1}(\tan b / \tanh a)
$$

(b) $\cosh (a+j b)=\cosh a \cdot \cos b+j \sinh a \cdot \sin b$

$$
=\sqrt{\sinh ^{2} a+\cos ^{2} b}\left(\tan ^{-1}(\tan b \cdot \tanh a)\right.
$$

(c) $\tanh (a+j b)=\sinh (a+j b) / \cosh (a+j b)$

$$
=\sqrt{\frac{\sinh ^{2} a+\sin ^{2} b}{\sinh ^{2} a+\cos ^{2} b}} / \tan ^{-1}\left(\frac{\sin 2 b}{\sin 2 a}\right)
$$

(d) $\tanh ^{-1}(a+j b)=x+j y$

Here, $x=\frac{1}{2} \log _{e} R, \quad y=\frac{\theta}{2}$
and $\frac{1+(a+j b)}{1-(a+j b)}=R / \theta$
(1) $\operatorname{Sinh}(a+j b)$

Example 24. $\sinh (0.43+j 0.68)=0.769 \quad 11.106$ $\sinh 0.43=0.443$

Move hairline to 0.43 on $\mathrm{Sh}_{1}$, under hairline find 4.43 on D , read answer as 0.443 .
$\sqrt{\sinh ^{2} 0.43+\sin ^{2} 0.68}=\sqrt{0.443^{2}+\sin ^{2} 0.68}=0.769$ Opposite 4.43 on P , set left index of Q , move hairline to 0.68 on Sr , under hairline find 7.69 on P , read answer as 0.769 .
$\tan ^{-1}\left(\frac{\tan 0.68}{\tanh 0.43}\right)=1.106$
Move hairline to 0.68 on $\mathrm{Tr}_{1}$, set 0.43 on $T h$ under hairline, move hairline to left index of C , under hairline read answer as 1.106 on $\mathrm{Tr}_{2}$.
(2) $\operatorname{Cosh}(a+j b)$

Example 25. $\operatorname { c o s h } ( 0 . 7 5 - j 1 . 2 4 ) = 0 . 8 8 4 \longdiv { 1 . 0 7 5 }$ $\sinh 0.75=0.822$
Move hairline to 0.75 on $\mathrm{Sh}_{1}$, under hairline find 8.22 on D , read answer as 0.822 .
$\sqrt{\sinh ^{2} 0.75+\cos ^{2} 1.24}=\sqrt{0.822^{2}+\cos ^{2} 1.24}=0.884$
Move hairline to right index of P , set 1.24 on Sr under hairline, move hairline to 8.22 on $Q$, under bairline find 8.84 on $P$, read answer as 0.884 .
$\tan ^{-1}(-\tan 1.24 \cdot \tanh 0.75)=-1.075$
Move hairline to 1.24 on $\mathrm{Tr}_{2}$,
set right index of C under hairline,
move hairline to 0.75 on Th ,
under hairline find 2.075 on $\mathrm{Tr}_{2}$,
read answer as -1.075 .
(3) $\mathrm{Tanh}^{-1}(a+j b)$

Example 26. $\tanh ^{-1}(0.2+j 0.4)=0.1735+j 0.393$ $\frac{1+(0.2+j 0.4)}{1-(0.2+j 0.4)}=\frac{1.2+j 0.4}{0.8-j 0.4}=\frac{\sqrt{1.2^{3}+0.4^{2}}}{\sqrt{0.8^{2}+0.4^{2}} / \tan ^{-1}(0.4 / 1.2)}$ $\sqrt{1.2^{2}+0.4^{2}}=1.265$
Move hairline to 12 on $P^{\prime}$, set left index of $Q$ under hairline, move hairline to 4 on $Q$,
under hairline find 12.65 on $P^{\prime}$,
read answer 1.265 .
$\tan ^{-1}\left(\frac{0.4}{1.2}\right)=0.322$
Move hairline to 4 on $D$,
set 12 on $C$ under hairline,
move hairline to left index of $C$,
under hairline read answer as 0.322 on $\mathrm{Tr}_{1}$.
$\sqrt{0.8^{2}+0.4^{2}}=0.894$
Opposite 8 on $P$, set left index of $Q$,
move hairline to 4 on $Q$,
under hairline find 8.94 on $P$,
read answer as 0.894 .
$\tan ^{-1}\left(\frac{-0.4}{0.8}\right)=-0.464$
Move hairline to 4 on D ,
set 8 on $C$ under hairline,
move hairline to right index of $C$,
under hairline find 0.464 on $\mathrm{Tr}_{\mathrm{r}}$,
read anwer as -0.464 .
$\therefore \frac{1.265 / 0.322}{0.894 /-0.464}=\frac{1.265}{0.894} / 0.322+0.464=1.415 / 0.786$

$$
x=\frac{I}{2} \log _{e} 1.415=\frac{1}{2} \times 0.347=0.1735
$$

Move hairline to 1.415 on $\mathrm{LL}_{2}$,
set 2 on $C$ under hairline,
opposite left index of $C$, find 1.735 on $D$,
read answer as 0.1735.
$y=\frac{\theta}{2}=\frac{0.786}{2}=0.393$
Thus the answer is;

$$
\tanh ^{-1}(0.2+j 0.4)=0.1735+j 0.393
$$

## 7. DECIBEL CALCULATION

In the electric communication circuit, let voltage and current at input side be $V_{1}$ and $I_{1}$, and those at output side be $V_{2}$ and $I_{2}$, the decibel for voltage ratio $d b_{(r}$, and the decibel for current ratio $d b_{(Y)}$ are as follows:

$$
\begin{aligned}
& d b_{(r)}=20 \log _{10} \frac{V_{3}}{V_{2}} \\
& d b_{(r)}=20 \log _{10} \frac{I_{2}}{I_{1}}
\end{aligned}
$$

When $V_{2}, V_{1}$ or $I_{2}, I_{1}$ are given, the ratio $\frac{V_{2}}{V_{1}}$ or $\frac{I_{2}}{I_{1}}$ is obtained by using the $C$ and $D$ scales, and the decibel of the ratio $\frac{V_{1}}{V_{1}}$ or $\frac{I_{2}}{I_{1}}$ is read on the db scale.

## 8. SOME APPLICATIONS ON ELECTRICAL PROBLEMS

Example 27. Calculate the current in an electric circuit, which impedance is $4+j 2.6$ and the potential difference between its terminals is $5+j 9$. Answer $i=1.907+j 1.013$

$$
\begin{aligned}
& \dot{I}=\frac{\dot{E}}{\dot{Z}}=\frac{5+j 9}{4+j 2.6}=\frac{\sqrt{5^{2}+9^{2}} / \tan ^{-1}\left(\frac{9}{5}\right)}{\sqrt{4^{2}+2.6^{2}} / \tan ^{-1}\left(\frac{2.6}{4}\right)} \\
& =\frac{10.3 / 1.064}{4.77 / 0.576} \\
& =\frac{10.3}{4.77} / 1.064-0.576 \\
& =2.16 / 0.488 \\
& \therefore \quad=2.16: \overline{\cos 0.488+j 2.16 \times \sin 0.488} \\
& =2.16 \times 0.883+j 2.16 \times 0.469 \\
& j=1.907+j 1.013
\end{aligned}
$$

Example 28. Compute the resultant current $\dot{I}$, of $\dot{I_{1}}=2+j 3$ and $\dot{I_{2}}=3+j 4$ in polar coodinate. Answer $i=8.60 / 0.95$

$$
\begin{aligned}
\dot{I}=\dot{I}_{1}+\dot{I}_{2} & =(2+j 3)+(3+j 4)=5+j 7 \\
& =\sqrt{5^{2}+7^{2}} / \tan ^{-2}\left(\frac{7}{5}\right) \\
\therefore \dot{I} & =8.60 / 0.95
\end{aligned}
$$

## 9. HOW TO USE LL SCALES

(1) Explanation of IL Scales

L工 represent that the scale is a logarithm of a logarithm. There are two groups of LL scales. One is $\mathrm{LI}_{n}$ scales ( $\mathrm{LL}_{2}$ and $\mathrm{LL}_{5}$ ) ranging from 1.105 to 20000 for the computation of the type form of $a^{+n}$ and the other is $L_{L_{u n}}$ scales ( $L L_{01}$, $L_{02}$ and ${L L_{03}}$ ) called Reciprocal $L_{L_{11}}$ scales ranging from 0.00005 to 0.99 for the computation of the type form of $a^{-n}$.
(2) Natural Logarithms

Set the hairline to the given number $N$ on the LL scale, $\log _{\varepsilon} N$ will be found out under the hairline on the $D$ scale.

Determining the position of the decimal point is as follows:
$\left.\begin{array}{l}\text { When } N \text { the } L L_{s} \text { scale } \\ \text { is set on the } L L_{3} \text { scale }\end{array}\right\} \cdots\left\{\begin{array}{l}\text { one digit at the left } \\ \text { one digit at the right }\end{array}\right\} \begin{aligned} & \text { of the de- } \\ & \text { cimal point, }\end{aligned}$ is set on the $L L L_{2}$ scale ${ }^{\cdots}$ one digit at the right $\begin{gathered}\text { cimal point, } \\ \text { of the de- }\end{gathered}$ when $N\left(\begin{array}{l}\text { the } L L_{03} \text { scale } \\ \text { is set on } \\ \text { the } L L_{02} s c a l e \\ \text { the } L L_{01} \text { scale }\end{array}\right) \quad \cdots$ one digit at the right $\begin{aligned} & \text { one digits at the right } \\ & \text { one left } \\ & \text { cimal point } \\ & \text { and pegace } \\ & \text { tive siga } \\ & \text { before the } \\ & \text { figures. }\end{aligned}$

Example 29. $\log _{e} 5=1.609$
Move hairline to 5 on $\mathrm{LL}_{3}$,
under hairline read answer as 1.609 on D.
Example 30. $\log _{e} 2=0.693$
Move hairline to 2 on $\mathrm{LL}_{2}$,
under hairline find 6.93 on $D$,
read answer as 0.693 .
Example 31. $\log _{e} 0.23=-1.47$
Move hairline to 0.23 on $\mathrm{LI}_{03}$,
under hairline find 1.47 on $D$,
read answer as -1.47 .
Example 32. $\log _{e} 0.955=-0.0461$
Move hairline to 0.955 on $\mathrm{LL}_{01}$, under hairline find 4.61 on $D$, read answer as -0.0461 .
(3) Powers and Roots

The type form of $a^{ \pm^{n}}$ or $a^{ \pm \frac{1}{n}}$ is simply calculated by the use ${ }^{-}$of the LL scales in an operation similar to multiplication and division.

Example 33. $4.25^{\cdot 12}=21.5, \quad 4.25^{-2.12}=0.0466$
Move hairline to 4.25 on $\mathrm{LL}_{3}$,
set left index of $C$ under hairline,
move hairline to 2.12 on C ,
under hairline read answer as 21.5 on $L_{L_{3}}$, under hairline read answer as 0.0466 on $L L_{03}$. Example 34. $1.96^{2 \cdot 3}=4.70, \quad 1.96^{-2 \cdot 30}=0.213$ Move hairline to 1.96 on $\mathrm{LL}_{2}$, set right index of $C$ under hairline, move hairline to 2.3 on $C$, under hairline read answer as 4.70 on $\mathrm{LL}_{3}$, under hairline read answer as 0.213 on $\mathrm{LL}_{03}$. Example 35. $11.4^{0.7}=5.50, \quad 11.4^{-0.7}=0.182$ Move hairline to 11.4 on $\mathrm{LL}_{3}$, set 7 on CI under hairline, move hairline to left index of $C$, under hairline read answer as 5.50 on $\mathrm{LL}_{3}$, under hairline read answer as 0.182 on $\mathrm{LL}_{03}$.
Example 36. $0.795^{1 \cdot 4}=0.725, \quad 0.795^{-1 \cdot 1}=1.379$
Move hairline to 0.795 on $\mathrm{LL}_{02}$, set left index of $C$ under hairline, move hairline to 1.4 on $C$,
under hairline read answer as 0.725 on $\mathrm{LL}_{\mathrm{j} 2}$. under hairline read answer as 1.379 on $\mathrm{LL}_{2}$.
Example 37. $e^{1.36}=7.10, e^{-1.96}=0.1408$
Move hairline to 1.96 on D , under hairline read answer as 7.10 on ${L L_{3}}$, under hairline read answer as 0.1408 on $\mathrm{LL}_{39}$.
Example 38. $\quad e^{0.91}=2.56, \quad e^{-0.94}=0.39$
Move hairline to 9.4 on D ,
under hairline read answer as 2.56 on $\mathrm{LL}_{2}$,
under hairline read answer as 0.39 on $\mathrm{LL}_{03}$.
(4) How to Use Lh. ' Scale

As an extension of the $C$ scale there is a very short
scale at the right end of the C scale marked from 1 to 1.1 in red. This scale called $L L_{1}{ }^{\prime}$ is invented in our laboratory to substitute the several lower LL scales in calculation of $\log _{e} N$ and $e^{x}$ etc.
(Natural Logarithms)
When $x$ is nearly equal to 1 , its natural logarithms are approximately equal to $x-1$. But, using the $\mathrm{LL}_{1}$ ' scale we can obtain its value more precisely. The procedure is:

Move the hairline to ( $x-1$ ) on the D scale, set $x$ on the $L_{1}{ }^{\prime}$ scale under the hairline, opposite the right index of the C scale, read the answer on the D scale.

Example 39. $\log _{6} 1.05=0.0488$
Move hairline $0.05(=1.05-1)$ on $D$,
set 1.05 on $\mathrm{LL}_{1}{ }^{\prime}$ under hairline,
opposite right index of C , find 4.88 on D ,
read answer as 0.0488 .
( $e^{x}$ )
When $x$ is nearly equal to zero, the value of $e^{x}$ is ap-
 obtain is precisely :

Opposite $z$ on the $D$ scale, set the right index of the $C$ scale, move the hairline to $(1+x)$ on the $L L_{1}{ }^{\prime}$ scale, under the hairline read the fractional part of the answer.

Example 40. $\quad e^{0.095}=1.0995$
Opposite 9.5 on D , set right index of C ,
move hairline to 1.095 on $\mathrm{LL}_{2}{ }^{\prime}$,
under hairline find 9.95 on D ,
read answer as $1+0.0995=1.0995$.

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