

BAMBOO SLIDE RULES

INSTRUCTIONS for the Use of

MODEL F-686

LAFAYETTE RADIO ELECTRONICS CORPORATION

Instrument Division 165-08 Liberty Avenue Jamaica 33, N. Y.

Printed in Japan

CONTENTS

INSTRUCTIONS

FOR THE USE OF F-686 SLIDE RULE

1.	G.	ENERAL DESCRIPTION 1
2.	S	CALE ARRANGEMENT AND USAGE 2
3.	\mathbf{T}	RIGONOMETRIC FUNCTIONS
	(1)	$\sin \theta$
	(2)	Cos $ heta$
	(3)	Conversion between Degree and Radian
	(4)	an heta
4.	V.	ECTOR PROBLEMS 5
	(1)	The Absolute Value of Vector
	(2)	Phase Angle of Vector
	(3)	Conversion of Coordinate System
	(4)	Multiplication and Division of Vectors
5.	H.	YPERBOLIC FUNCTIONS 8
	(1)	Sinh x
	(2)	Tanh x
	(3)	Cosh #
6.	\mathbf{H}	PERBOLIC FUNCTIONS OF COMPLEX
	Į	ANGLES
	(1)	Sinh $(a+jb)$
	(2)	$\cosh(a+jb)$
	(3)	$Tanh^{-1}(a+jb)$
7.	DE	CIBEL CALCULATION12
8.	SO	ME APPLICATIONS ON ELECTRICAL
	P	ROBLEMS12
9.		W TO USE LL SCALES
		Explanation of LL Scales
	-	Natural Logarithms
		Powers and Roots
	• •	How to Use LL ₁ ' Scale

Rec. to Polar

Opsite the larger no. on P set the left index of Q. Move hair line to smaller no. on Q read absolute value on P. Move hair line to smaller no. on D. On C set larger No. under hair line Move Hair line to right index of C and read Radan of Angle on TR-1. If right index is off scale use left index and read TR-2. Convert Radan to Degree

INSTRUCTIONS

FOR THE USE OF F-686 SLIDE RULE

1. GENERAL DESCRIPTION

This slide rule has been designed for expert electrical engineer to simplify the various calculations occurred frequently in electricity, namely not only the computation of multiplication and division can be done with the A, B, C, D, CI, DF and CF scales, but also the LL₈, LL₂, LL₁', LL₀₈, LL₀₂ and LL₀₁ scales make it possible to obtain the result of a^n , e^x and log, N. Moreover, the P, Q and P' scales are essential for vector computation, the Sh₂, Sh₁ and Th scales for hyperbolic function.

2. SCALE ARRANGEMENT AND USAGE

- (Front Face) Tr_1 , Tr_2 , P', P, Q, ST, Sr, S θ , C, D, LL_{03} , LL_{03} , LL_{01}
- (Back Face) Sh₁, Sh₂, DF, A, B, CF, Th, CI, C, D, LL₂, LL_3 , db, LL_1' .

(a) Tri and Tri

These scales are used in the computation of the tangent of an angle from 0.1 to 0.8 radians, and from 0.8 to 1.472 radians respectively.

(b) P, Q and P'

The computation of the vector can be conveniently done by these scales as ordinary multiplication and division done by the C and D scales.

(c) ST

This scale gives the sine and tangent of an angle less than 6° .

(d) Sr and S θ

These scales are used to obtain the sine and cosine of an angle, co-operate with the P and Q scales. Angles are graduated at degree and its decimal fraction in the Sr scale, and at radian in the S θ scale. Thus conversion from degree to radian or its reverse process is made by the use of these scales.

(e) LLos, LLos and LLos

These are called the reciprocal Log Log scales and are used to find the values of the type form of a^{-n} , e^{-x} and $\log_e N$.

(f) Shi and Shi

These scales make it possible to compute the sine-hyperbolic function, which is frequently necessary in alternating current theory.

(g) Th

This gives the value of the tangent-hyperbolic function.

(h) LLs, LLz and LL1'

The LL₃ and LL₄ are called the Log Log scales and are used to find the values of the type form of a^n , e^x and $\log_e N$.

 LL_1' , which as an extension scale of C, is marked in red at the right end of it, substitutes the ordinary LL_1 scale. (i) db

This decibel scale is useful in the computation of electric communication circuit. Moreover this equaly subdivided scale can be used as ordinary mantissa scale to obtain the common logarithms of a number.

(j) A, B, C, D, Cl, DF and CF

Of these scales, the DF and CF scales are the D and C scales folded at π , so every number on these scales is equal to the number on the D and C scales multiplied by π .

3. TRIGONOMETRIC FUNCTIONS

(1) Sin θ

The sine of an angle, either in degree or in radian can be directly read on S θ or Sr scale. Example 1. $\sin 51.8^{\circ} = 0.786$ Move hairline to 51.8 on S θ . under hairline find 7.86 on Q. read answer as 0.786. Example 2. sin 0.665=0.617 Move hairline to 0.665 on Sr. under hairline find 6.17 on Q. read answer as 0.617. The value of the sine or tangent of an angle less than 6° is calculated by the use of the ST scale. Example 3. $\sin 1.5^{\circ} = 0.0262$ Move hairline to 1.5 on ST. under hairline find 2.62 on C, read answer as 0.0262. (2) $\cos \theta$ Example 4. $\cos 51.8^\circ = 0.618$ Move hairline to right index of P, set 51.8 on S θ under hairline, opposite left index of Q find 6.18 on P, read answer as 0.618. Example 5. $\cos 0.665 = 0.787$

4

Move hairline to right index of P, set 0.665 on Sr under hairline, opposite left index of Q find 7.87 on P, read answer as 0.787.

(3) Conversion between Degree and Radian

Conversion from degree to radian or its reverse process is made by using the Sr and S θ scales.

Example 6. $53^\circ = 0.925$ radians

Move hairline to 53 on S θ ,

under hairline read answer as 0.925 on Sr.

Example 7. 0.52 radians=29.8°

Move hairline to 0.52 on Sr,

under hairline read answer as 29.8 on S θ .

In the case of conversion of a small angle, move the decimal point one place to the right, perform the oparation as is explained above, and read the answer moving the decimal point one place lower.

Example 8. 0.06 radians= 3.44° Move hairline to 0.6 on Sr, under hairline find 34.4 on S θ , read answer as 3.44.

(4) Tan θ

The Tr_1 and Tr_2 scales represent a single scale of angles ranging from 0.1 to 1.472 radians.

When using the Tr_1 scale to read the value of tan θ , read the left index of C as 0.1 and the right index as 1, and using the Tr_2 scale to read the value of tan θ , read the left index of C as 1 and the right index as 10.

Example 9. tan 0.35=0.365 Move hairline to 0.35 on Tr. under hairline find 3.65 on D, read answer as 0.365. Example 10. tan 1.17=2.36 Move hairline to 1.17 on Tr₂, under hairline read answer as 2.36.

4. VECTOR PROBLEMS

(1) The Absolute Value of Vector

The absolute value of vector, represented in the type form of a+jb, is equal to $\sqrt{a^2+b^2}$, and by the use of the P, Q and P' scales this value can be computed very easily in the same operation as ordinary multiplication and division. Namely set zero on the Q scale to a on the P scale, opposite b on the Q scale read $\sqrt{a^2+b^2}$ on the P scale. When b on the Q scale runs off the P scale, then set 10 on the Q scale to a on the P scale, and opposite b on the Q scale read the answer on the P' scale.

Example 11. Find the absolute value of 3+j4. Opposite 3 on P, set left index of Q, move hairline to 4 on Q, under hairline read answer as 3^{\prime} on P. Example 12. $\sqrt{8.76^2+9.32^2}=12.79$ Opposite 8.76 on P, set right index of Q, move hairline to 9.32 on Q, under hairline read answer as 12.79 on P'. The computation of the type form of $\sqrt{a^2-b^2}$ can be done in the same way. Example 13. $\sqrt{10.53^2-5.96^2}=8.68$ Move hairline to 10.53 on P', set 5.96 on Q under hairline, 6

opposite right index of Q, read answer as 8.68 on P. Example 14. $\sqrt{93.4^2 - 41.8^2} = 83.5$ Calculate as $10 \times \sqrt{9.34^2 - 4.18^2}$ move hairline to 9.34 on P. set 4.18 on Q under hairline. opposite left index of Q, find 8.35 on P. read answer as $10 \times 8.35 = 83.5$. (2) Phase Angle of Vector In the vector of the type form of a+ib, the phase angle θ is represented as follows: $\theta = \tan^{-1} \frac{b}{a}$ Example 15. $\tan^{-1}\frac{3.6}{2.5} = 0.964$ Move hairline to 3.6 on D. set 25 on C under hairline. move hairline to left index of C. under hairline read 0.964 on Tra-Example 16. $\tan^{-1} \frac{2.5}{3.6} = 0.607$ Move hairline to 2.5 on D. set 3.6 on C under hairline. move hairline to right index of C. under hairline read 0.607 on Tr. (3) Conversion of Coordinate System From the following relations: a+ ih= 1/22 1 10/ 10-1 b

$$here \quad R = \sqrt{a^2 + b^2} / \frac{\tan^{-1} - R}{a} = R / \frac{\theta}{a}$$
$$\theta = \tan^{-1} \frac{b}{a}$$

$R_{i}^{\prime}\theta = R \cdot \cos \theta + jR \cdot \sin \theta$

the conversion of vector in polar coordinate system to rectangular coordinate system and its reverse computation can be easily done.

Example 17. $-7.5+j6.0=9.6/\pi-0.675$

This is the example of conversion of a+jb to polar coordinate system.

Opposite 7.5 on P, set left index of Q, move hairline to 6.0 on Q, under hairline read absolute value as 9.6 on P. Move hairline to 6.0 on D, set 7.5 on C under hairline, move hairline to right index of C, under hairline read 0.675 on Tr_1 , thus the phase angle is equal to π -0.675. Example 18. $25/52^\circ = 15.4 \pm j$ 19.7 This is the example of conversion of polar coordinate

system to a+jb.

Move hairline to right index of P, set 52 on S θ under hairline, under hairline read sin 52°=0.788 on Q, opposite left index of Q, read cos 52°=0.616 on P, thus, real part is calculated as 25×0.616=15.8 and imaginary part j 25×0.788=j 19.7.

(4) Multiplication and Division of Vectors

Multiplication and Division of two vectors have been computed from the following formulas:

 $\frac{R_1/\theta_1}{R_2/\theta_2} \times \frac{R_2}{R_1} = \frac{R_1}{R_2} \times \frac{R_2}{R_1} + \frac{R_2}{R_2} = \frac{R_1}{R_2} + \frac{R_2}{R_2} + \frac{R_1}{R_2} + \frac{R_2}{R_2} + \frac{R_2}$

Example 19.
$$\frac{3-j2}{8+j4} = 0.404\overline{1.052}$$
$$\frac{3-j2}{8+j4} = \frac{\sqrt{3^2+2^2}/\tan^{-1}\left(\frac{-2}{3}\right)}{\sqrt{8^2+4^4}/\tan^{-1}\left(\frac{4}{8}\right)}} = \frac{3.61/-0.588}{8.94/0.464}$$
$$= 0.404/-0.588-0.464$$
$$= 0.404/-1.052 = 0.404\overline{1.052}$$

5. HYPERBOLIC FUNCTIONS

The logarithmic scales Sh_1 , Sh_2 and Th are used for the computation of hyperbolic function.

(1) Sinh x

8

The Sh₁ and Sh₂ scales give the value of sinh x. When using the Sh₁ scale to read the value of sinh x, read the left index of D as 0.1 and the right index as 1, and using the Sh₂ scale to read the value of sinh x, read the left index of D as 1 and the right index as 10.

Example 20. sinh 0.362=0.370 Move hairline to 0.362 on Sh₁, under hairline find 3.70 on D, read answer as 0.370.

Example 21. $\sinh 2.56 = 6.42$

Move hairline to 2.56 on Sh₂,

under hairline read answer as 6.42 on D.

(2) Tanh x

The Th scale gives the value of tanh x. When using the Th scale to read the value of tanh x, read the left index of C as 0.1 and the right index as 1.

Example 22. $\tanh 0.183 = 0.181$ Move hairline to 0.183 on Th. under hairline find 1.81 on C, read answer as 0.181.

(3) Cosh x

(1

The value of $\cosh x$ can be computed from the following formula:

$$\cosh x = \frac{\sinh x}{\tanh x}$$

Example 23. cosh 0.575=1.170 Move hairline to 0.575 on Sh₁, set 0.575 on Th under hairline, opposite left index of C read answer as 1.170 on D.

6. HYPERBOLIC FUNCTIONS OF COMPLEX ANGLES

Hyperbolic functions of complex angles are given from the following formulas:

(a)
$$\sinh (a+jb) = \sinh a \cos b + j \cosh a \sin b$$

$$= \sqrt{\sinh^2 a + \sin^3 b} / \frac{\tan^{-1}(\tan b/\tanh a)}{(\tan^{-1}(\tan b/\tanh a))}$$
(b) $\cosh (a+jb) = \cosh a \cdot \cos b + j \sinh a \cdot \sin b$

$$= \sqrt{\sinh^2 a + \cos^2 b} / \frac{\tan^{-1}(\tan b \cdot \tanh a)}{(\tan^{-1}(\tan b \cdot \tanh a))}$$
(c) $\tanh (a+jb) = \sinh (a+jb)/\cosh (a+jb)$

$$= \sqrt{\frac{\sinh^2 a + \sin^2 b}{\sinh^2 a + \cos^3 b}} / \frac{\tan^{-1}(\frac{\sin 2 b}{\sin 2 a})}{(\sin 2 a)}$$
(d) $\tanh^{-1}(a+jb) = x+jy$
Here, $x = \frac{1}{2} \log_e R$, $y = \frac{\theta}{2}$
and $\frac{1+(a+jb)}{1-(a+jb)} = R/\theta$
) $\sinh (a+jb)$
frample 24 $\sinh (0.43 + i0.68) = 0.760 / 1.106$

Example 24. $\sinh (0.43 + j0.68) = 0.769 / 1.106$ $\sinh 0.43 = 0.443$ 10

Move hairline to 0.43 on Sh₁, under hairline find 4.43 on D. read answer as 0.443. $\sqrt{\sinh^2 0.43 + \sin^2 0.68} = \sqrt{0.443^2 + \sin^2 0.68} = 0.769$ Opposite 4.43 on P. set left index of Q. move hairline to 0.68 on Sr, under hairline find 7.69 on P, read answer as 0.769. $\tan^{-1}\left(\frac{\tan 0.68}{\tanh 0.43}\right) = 1.106$ Move hairline to 0.68 on Tr., set 0.43 on Th under hairline, move hairline to left index of C, under hairline read answer as 1.106 on Tr., (2) Cosh (a+jb)Example 25. $\cosh(0.75 - i1.24) = 0.884 \sqrt{1.075}$ $\sinh 0.75 = 0.822$ Move hairline to 0.75 on Sh., under hairline find 8.22 on D. read answer as 0.822. $\sqrt{\sinh^2 0.75 + \cos^2 1.24} = \sqrt{0.822^2 + \cos^2 1.24} = 0.884$ Move hairline to right index of P, set 1.24 on Sr under hairline. move hairline to 8.22 on Q. under hairline find 8.84 on P, read answer as 0.884. $\tan^{-1}(-\tan 1.24 \cdot \tanh 0.75) = -1.075$ Move hairline to 1.24 on Tr_{2} , set right index of C under hairline move hairline to 0.75 on Th. under hairline find 1.075 on Tr_2 ,

read answer as -1.075. (3) Tanh⁻¹ (a+jb)Example 26. $\tanh^{-1}(0.2+j0.4) = 0.1735+j0.393$ $\frac{1+(0.2+j0.4)}{1-(0.2+j0.4)} = \frac{1.2+j0.4}{0.8-j0.4} = \frac{\sqrt{1.2^2+0.4^2}}{\sqrt{0.8^2+0.4^2}} \frac{/\tan^{-1}(0.4/1.2)}{/\tan^{-1}(-0.4/0.8)}$ $\sqrt{1.2^2 + 0.4^2} = 1.265$ Move hairline to 12 on P', set left index of Q under hairline, move hairline to 4 on Q. under hairline find 12.65 on P', read answer 1.265. $\tan^{-1}\left(\frac{0.4}{1.2}\right) = 0.322$ Move hairline to 4 on D, set 1.2 on C under hairline, move hairline to left index of C, under hairline read answer as 0.322 on Tr₁. $\sqrt{0.8^2+0.4^2}=0.894$ Opposite 8 on P, set left index of Q, move hairline to 4 on Q, under hairline find 8.94 on P, read answer as 0.894. $\tan^{-1}\left(\frac{-0.4}{0.8}\right) = -0.464$ Move hairline to 4 on D, set 8 on C under hairline, move hairline to right index of C, under hairline find 0.464 on Tr,, read answer as -0.464. $\frac{1.265 \ /0.322}{0.894 \ /-0.464} = \frac{1.265}{0.894} \ /0.322 + 0.464 = 1.415 \ /0.786$

$$x = \frac{1}{2} \log_{e} 1.415 = \frac{1}{2} \times 0.347 = 0.1735$$

Move hairline to 1.415 on LL₂,
set 2 on C under hairline,
opposite left index of C, find 1.735 on D,
read answer as 0.1735.
$$y = \frac{\theta}{2} = \frac{0.786}{2} = 0.393$$

Thus the answer is;

 $tanh^{-1}(0.2+j0.4) = 0.1735+j0.393$

7. DECIBEL CALCULATION

In the electric communication circuit, let voltage and current at input side be V_1 and I_1 , and those at output side be V_2 and I_2 , the decibel for voltage ratio $db_{(1)}$ and the decibel for current ratio $db_{(1)}$ are as follows:

$$db_{(V)} = 20 \log_{10} \frac{V_2}{V_1}$$
$$db_{(I)} = 20 \log_{10} \frac{I_2}{I_1}$$

When V_2 , V_1 or I_2 , I_1 are given, the ratio $\frac{V_2}{V_1}$ or $\frac{I_2}{I_1}$ is obtained by using the C and D scales, and the decibel of the ratio $\frac{V_2}{V_1}$ or $\frac{I_2}{I_1}$ is read on the db scale.

8. SOME APPLICATIONS ON ELECTRICAL PROBLEMS

Example 27. Calculate the current in an electric circuit, which impedance is 4+j2.6 and the potential difference between its terminals is 5+j9. Answer I=1.907+j1.013

$$\begin{split} \vec{I} &= \frac{\vec{E}}{\vec{Z}} = \frac{5+i9}{4+j2.6} = \frac{\sqrt{5^2+9^2}}{\sqrt{4^2+2.6^2}} \frac{1}{\tan^{-1}\left(\frac{9}{5}\right)} \\ &= \frac{10.3}{\sqrt{4^2+2.6^2}} \frac{1}{\tan^{-1}\left(\frac{2.6}{4}\right)} \\ &= \frac{10.3}{4.77} \frac{1.064}{10.64} \\ &= \frac{10.3}{4.77} \frac{1.064-0.576}{10.64-0.576} \\ &= 2.16 \frac{10.488}{10.488} \\ \therefore &= 2.16 \times \cos 0.488 + j2.16 \times \sin 0.488 \\ &= 2.16 \times \sin 0.488 + j2.16 \times \sin 0.488 \\ &= 2.16 \times \sin 0.488 + j2.16 \times \sin 0.488 \\ &= 2.16 \times \sin 0.488 + j2.16 \times \sin 0.488 \\ &= 2.16 \times \sin 0.488 + j2.16 \times \sin 0.488 \\ &= 2.16 \times \sin 0.488 + j2.16 \times \sin 0.488 \\ &= 2.16 \times \sin 0.488 + j2.16 \times \sin 0.488 \\ &= 2.16 \times \sin 0.488 + j2.16 \times \sin 0.488 \\ &= 2.16 \times \sin 0.488 + j2.16 \times \sin 0.488 \\ &= 2.16 \times \sin 0.488 + j2.16 \times \sin 0.488 \\ &= 2.16 \times \sin 0.488 + j2.16 \times \sin 0.488 \\ &= 2.16 \times \sin 0.488 + j2.16 \times \sin 0.4$$

(1) Explanation of LL Scales

LL represent that the scale is a logarithm of a logarithm. There are two groups of LL scales. One is LL_n scales (LL_2 and LL_s) ranging from 1.105 to 20000 for the computation of the type form of a^{+n} and the other is LL_{on} scales (LL_{01} , LL_{o2} and LL_{03}) called Reciprocal LL_n scales ranging from 0.00005 to 0.99 for the computation of the type form of a^{-n} .

(2) Natural Logarithms

Set the hairline to the given number N on the LL scale, log, N will be found out under the hairline on the D scale.

Determining the position of the decimal point is as follows:

When N the LL₃ scale \dots one digit at the left one digit at the right of the decimal point, of the de-(the LL_{03} scale) (one digit at the left) cimal point place when Nand the LL_{02} scale ... one digit at the right the negative sign is set on two digits at the right before the the LL_{01} scale) figures. Example 29. log. 5=1.609 Move hairline to 5 on LL_{*}. under hairline read answer as 1.609 on D. Example 30. log, 2=0.693 Move hairline to 2 on LL₂, under hairline find 6.93 on D. read answer as 0.693. Example 31. $\log_{e} 0.23 = -1.47$ Move hairline to 0.23 on LLos, under hairline find 1.47 on D, read answer as -1.47. Example 32. $\log_{e} 0.955 = -0.0461$ Move hairline to 0.955 on LL₆₁, under hairline find 4.61 on D, read answer as -0.0461. (3) Powers and Roots The type form of $a^{\pm n}$ or $a^{\pm \frac{1}{n}}$ is simply calculated by the use of the LL scales in an operation similar to multiplication and division. Example 33. $4.25^{2 \cdot 12} = 21.5$, $4.25^{-2 \cdot 12} = 0.0466$ Move hairline to 4.25 on LL₃, set left index of C under hairline,

move hairline to 2.12 on C,

under hairline read answer as 21.5 on LL. under hairline read answer as 0.0466 on LLos. Example 34. $1.96^{2 \cdot 3} = 4.70$, $1.96^{-2 \cdot 30} = 0.213$ Move hairline to 1.96 on LL₂, set right index of C under hairline, move hairline to 2.3 on C, under hairline read answer as 4.70 on LL₃, under hairline read answer as 0.213 on LL₀₃. Example 35. 11.4°*7=5.50, 11.4°*7=0.182 Move hairline to 11.4 on LL,, set 7 on CI under hairline, move hairline to left index of C, under hairline read answer as 5.50 on LL_a under hairline read answer as 0.182 on LL_{23} . Example 36. 0.795¹ = 0.725, 0.795⁻¹ = 1.379 Move hairline to 0.795 on LL₀₂, set left index of C under hairline, move hairline to 1.4 on C. under hairline read answer as 0.725 on LL₁₂. under hairline read answer as 1.379 on LL₂. Example 37. $e^{1.96} = 7.10$, $e^{-1.96} = 0.1408$ Move hairline to 1.96 on D, under hairline read answer as 7.10 on LL_i, under hairline read answer as 0.1408 on LL₁₃. Example 38. $e^{0.91} = 2.56$, $e^{-0.94} = 0.39$ Move hairline to 9.4 on D, under hairline read answer as 2.56 on LL₂, under hairline read answer as 0.39 on LL₀₂.

(4) How to Use LL1' Scale

As an extension of the C scale there is a very short

scale at the right end of the C scale marked from 1 to 1.1 in red. This scale called LL_1' is invented in our laboratory to substitute the several lower LL scales in calculation of log. N and e^x etc.

(Natural Logarithms)

When x is nearly equal to 1, its natural logarithms are approximately equal to x-1. But, using the LL₁' scale we can obtain its value more precisely. The procedure is:

Move the hairline to (x-1) on the D scale, set x on the LL_1' scale under the hairline, opposite the right index of the C scale, read the answer on the D scale.

Example 39. log, 1.05=0.0488 Move hairline 0.05(=1.05-1) on D, set 1.05 on LL₁' under hairline, opposite right index of C, find 4.88 on D, read answer as 0.0488.

(e*)

When x is nearly equal to zero, the value of e^x is approximately equal to (1+x). Using the LL₁' scale we can obtain is precisely:

Opposite x on the D scale, set the right index of the C scale, move the hairline to (1+x) on the LL_1' scale, under the hairline read the fractional part of the answer.

Example 40. $e^{0.095} = 1.0995$

Opposite 9.5 on D, set right index of C, move hairline to 1.095 on LL_1' , under hairline find 9.95 on D, read answer as 1+0.0995=1.0995.

This Page Blank